Media persuasion and voter welfare

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Abstract

We study information transmission where an informed media, whose interests are partially in conflict with a finite group of rational voters, transmits news items in an attempt to manipulate democratic decisions. In a common-interest two-alternative voting model where due to reputation concerns the media can credibly commit to send any news reliably, we show that even if voters welcome the news when it arrives, media’s presence can hurt their ex-ante welfare in both large and small constituencies.

Keywords Media · Voting · Welfare

JEL D60 · D71 · D72 · D82

1 Introduction

Voting is a popular institutional apparatus to aggregate information and take better social decisions. By making privately informed voters cast independent votes,
society can increase the probability of electing policy alternatives that are welfare enhancing for the voters, particularly when voters have common preferences. In such circumstances, the role of an informed media or expert forecaster can be decisive. Behind the cry for the right to information and freedom of press lies an argument that media keeps voters well aware of key social and economic variables so that their personal judgments are more informed. 

It is often the case that the media has its own biases that reflect preferences of a minority of the population. A survey by Lichter et al. (1986) suggests that in the decade from 1975 to 1985, conservative ideologies outnumbered liberals among the American public while during that period close to three quarters of journalists held strong liberal views. With the power to communicate publicly, one would expect an informed (but biased) media to indulge in deliberate manipulation of social decisions through instruments such as keeping the news vague or revealing the truth only if certain circumstantial evidence is available. This can make media advice partly unreliable. To this end, Watts et al. (1999) provides evidence about voter perception of biased media news during the 1988, 1992 and 1996 presidential elections in the US. The study reveals that there was a rise in public perception that the media was liberally biased and a leading cause of this was an increased participation of liberal elites in news items. Yet, the most influential media sources in democratic societies are professional organizations or individuals who care about their reputation as a reliable platform for public debates and news in the future. Hence even known biases of the media cannot eliminate entirely the credibility of the news when it is transmitted.

While public knowledge about the media’s reputation concerns makes the media better placed to credibly transmit any information in spite of its biases, this power of credibility in turn enhances its ability to manipulate voting outcomes. On the other hand if it is known that the media does not care about its reputation,

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1 Of course, media has other roles in public life that are also useful, such as keeping a check on corruption and crime. The present paper abstracts away from these issues.
it may not enjoy the same public trust as its reputation caring counterpart would. While the ability of media without reputational concerns to inform is then less, so also is its power to manipulate. It then remains ambiguous what is better for the voters: higher credibility (at a cost of higher ability to manipulate) or lower ability to manipulate (at a cost of lower credibility). Given this ambiguity, in this paper we ask what are the theoretical consequences of the presence of reputational concerns of the media on the welfare of the voters. In particular, we ask if the presence of reputation driven media transmitting credible news unambiguously improves ex-ante voter welfare. We show that the answer depends upon various aspects of the environment including size of the constituency, degree of conflict between the voters and the media and other informational parameters like priors and signal strengths of voters’ private information. In general, media presence tends to hurt when the constituency is large and voters receive strong private signals.

We study a society represented by an odd number of voters. The voters face uncertainty over the true state of the world and must choose collectively (via the majoritarian voting rule) between two alternatives, $X$ and $Y$. They have common preferences over states and alternatives and hold a common prior over the states. In addition, each voter receives private information about the state. The quality of this information is common and reflects the general degree of individual awareness in the society.\(^2\)

The media is an expert in the matter and knows the state, but has preferences that are not perfectly aligned with those of the voters. In particular, the media prefers $X$ in all states.\(^3\) The media is reputation driven and can credibly deliver any news if it is in its best interest to do so. Thereafter, voting takes place.

The benchmark model is normalized in a manner where without reputation, no information can be transmitted by the media (and all results we report below

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\(^2\)Individual awareness is typically reflected in the general level of education or other factors which influence the ability of a voter to analyze privately obtained information.

\(^3\)Our results remain qualitatively intact in a more general environment where in some states the media can prefer $Y$. 

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where the media is absent can also be interpreted as outcomes with a media that
does not care about future reputation). The resulting voting subgame resembles a
common interest jury environment. Under the simple-majoritarian rule, there is a
Bayes-Nash equilibrium (in weakly undominated strategies) where each voter votes
on the basis of his private signal. It is well known that if the precision of signals
is smaller than some cut-off value, pooling appears unambiguously in equilibrium
where all voters commonly vote for one of the alternatives with probability 1
as their judgments are overwhelmed by their (common) priors. Hence, the ex-
ante voter welfare remains invariant with the size of the constituency. When the
signal strength is higher than this cut-off, the equilibrium becomes fully separating
where an individual vote perfectly reveals the voter’s private signal. In that case,
as the size of the constituency increases, standard Condorcet Jury Theorem (CJT
henceforth) arguments come into play. Consequently, the voters’ ex-ante welfare
increases monotonically with the size of the constituency.

When reputed media (henceforth, media) is present, we look at equilibria that
maximize the media’s ex-ante payoff. These equilibria are typically informative
and can be thought of as ones where the media has the highest ideological influence
over the social decision. Here, by revealing information carefully, the media can
manipulate voting behavior to its own benefit. As in some states the two sides
hold conflicting preferences, this may hurt the voters. Yet, at the time voters
receive a piece of news, they find the news useful as privately they still remain
only partially informed of the true state and know that the informative content of
the news transmitted by the media can be relied upon.

In our analysis, we first consider the benchmark model with a single voter (or
decision maker, DM in short) and then proceed to the multi voter case. If the
media chooses to transmit informative news, then both in the case of a single DM
or in the multi-voter case, it will typically send slanted news that either endorses
its preferred policy $X$ or recommends against it. We show that with a single
DM, the presence of the media can never affect welfare adversely. This result is
however altered when multiple voters are considered. We show that regardless of the size of the constituency, media presence can adversely affect the ex-ante probability of a correct decision and hence voters’ welfare. If the prior bias of the voters is towards what the media wants (suggesting smaller conflict between the two sides), news will not be informative if voters private signals carry little information (i.e. an unaware society). As the voters individually get more aware of the uncertainties, informative news will be transmitted. Moreover, media presence will necessarily hurt voter welfare when the constituency is large. This perverse effect of informative news can appear in small constituencies as well and we provide some characterizations in this respect.

The outcomes change significantly when the prior bias of the voters is against what the media wants (suggesting larger conflict between the two sides). In this case for a sufficiently aware society, under certain scenarios the media transmits information such that any news slanted towards media’s bias will be fully revealing, while news with a slant against media’s own preference will be inconclusive. Interestingly, for these cases, the voters will follow the media slant only when the media endorses what it wants but vote according to their private signals when the news is slanted against the media’s bias. We show that for a sufficiently aware society, the presence of the media can both be beneficial and harmful for welfare in these cases. However, for a society where the level of awareness is low, we show that the presence of the media is always welfare improving.

These results suggest that valuable news from a biased media is always welcome when the social decision is taken by a single DM while as democracy spreads and the number of voters get large, a reliable source of news can transmit immiserizing information. It is important to note that while the possibility of immiserizing information in our model is driven by the presence of strategic incentives of the media, similar public information can appear from non-strategic actors as well or even from sources whose intention is to send as much information as possible under the belief that more information cannot hurt the voter. This is specially
true in the case of court trials where the judiciary tries to ensure that the trial is as informative as possible. Yet, since not all trials can reveal the truth with certainty, one may ask if and when any additional (but partial) public information adversely affects the probability of wrong judgments. Our results then indicate that when the trial is known to provide partially informative slants with evidential input, the probability that a jury takes the correct decision can go down, though such trials are always welcome if the jury is replaced by a dictatorial judge (single DM in our case). Moreover, when the jury is a priori biased towards one outcome (say acquittal) and the jury members receive strong private signals, using a jury may be better than a dictatorial judge. While a full analysis of these issues is beyond the scope of the present paper, this is an important avenue for future research.

1.1 Related literature

The social value of public information has been a well addressed subject since the work of Hirshleifer (1971). In a model with complementarities, Morris and Shin (2002) shows that public information can hurt social welfare while in the investment game of Angeletos and Pavan (2004) and in a monetary policy game of Hellwig (2005), public information necessarily improves welfare. Also, Angeletos and Pavan (2007) show how welfare properties of public information depends not only on the form of strategic interaction but also on other external effects that determine the gap between equilibrium and efficient use of public information.\footnote{See also Bikchandani et al. (1992), Cao and Hirshleifer (2000) and Gersbach (2000) among others for related works on impact of public information on social welfare.} However, in all these works, transmission of public information is non-strategic. Besides, to the best of our knowledge, ours is the first paper to address this issue in a strategic voting environment.

Our work is also closely related to two other strands of literature, namely, cheap talk and voting, and to an emerging area of research that links the two. The seminal work by Crawford and Sobel (1982) describes a framework of communica-
tion between an informed sender and an uninformed receiver (or decision maker) where messages are costless and do not directly affect the utility of the sender. However, we consider a particular setting where the reputation-driven sender (the media) can potentially transmit vague information but cannot lie. This is akin to ‘costly talk’ and is in accordance with the ‘verifiable disclosure’ approach to communication, initiated by Grossman (1981) where the sender cannot lie but can withhold information. Kartik et al. (2007) and Kartik (2009) have also modified the assumption of the sender’s messages being costless and have instead considered communication where the senders suffer disutility from lying. Our approach to modeling a reputation-driven sender has resemblance to Chen (2011), who considers a finite message space and defines an ‘honest’ sender to be one who by nature always reports the message that is closest to her observation.

Our model fundamentally differs from the above body of work in two aspects: First, we model a binary decision problem where the action space of the voters is finite and not continuous (as in the above papers). Second, we consider a single sender and multiple receivers with partially aligned interests, in the sense that there is a range of states of the world where the preferred choice of the receivers (voters) as well as the sender (media) is identical, whereas for some states they vary. In contrast in the above papers, there is conflict of interest in each state of the world.

Next, consider our results concerning certain scenarios where the information provided by the sender is such that the voters vote according to their private signals. In these instances the welfare analysis of the voters is related to the probability of correct decisions in the CJT literature. Austen-Smith and Banks (1996) study the innocuousness of the assumption made in the statement of CJT that the voters vote ‘sincerely’ (that is, even when they are in a group they vote as if alone) and introduces strategic voting. They show that ‘sincere’ voting, which implies voting in accordance to private signals is equivalent to strategic voting if the aggregation rule is simple majoritarian. For relatively high levels of precision of
private signals received by the voters, this result is obtained in the communication
game we study as well.\footnote{See also Feddersen and Pesendorfer (1996, 1998) for more on this issue.}

Our work is also very closely related to a relatively scant but growing literature
on how social elites (media, popular political and apolitical figures) can influence
mass opinions through public information (for some review articles on this, see
Mutz et al. (1996), Kinder (1998) and Druckman and Lupia (2000)). Iyengar
and Kinder (1987) and Lupia and McCubbins (1998) study the role of media in
persuading politically aware citizens to elect parties in democratic societies. Zaller
(1992) suggests related theories of how political awareness and greater cognitive
engagement of citizens with public sources of political news can affect political
opinions and support. The idea that the media may represent biased elitist opin-
ions regarding political choice and that voters ‘like’ the influence cast by such
political endorsements of the media is also addressed in Mullainathan and Shleifer
(2005). That political commentators can be reputation driven (because of career
concerns) has been noted in Gentzkow and Shapiro (2006) though they derive
biased media coverage without ideological considerations. Other works on politi-
cal economy and the role of the public endorsements (see for example Grossman
and Helpman (1999) and Stromberg (2004)) study how direct or indirect mass
communication with the voters can influence electoral outcomes.

Besley and Prat (2006) and Anderson and McLaren (2010) study how biased
media can communicate with voters using ex-post verifiable messages about qual-
ity of competing parties. In contrast Chakraborty and Ghosh (2012) studies a
cheap talk model where elite and partisan media can advertise about the quality
of two competing candidates in an otherwise Hotelling-Downs framework (with
sincere voting). In their paper, the ideological positions of different media outlets
make candidates strategically position themselves on the ideology line in order to
gain media attention. Voters are sophisticated enough to discount both the me-
dia endorsements and the pandering of parties towards the media outlets. Among
other characterizations of resulting electoral equilibria they demonstrate the possibility of situations where the voters are better off when there are no media outlets present even though their existence generate useful information transmission. The idea there is that while media reveal information about quality of competing parties, their existence in turn affects equilibrium policies in a way that ultimately can hurt social welfare. Our welfare analysis shows that this phenomenon of immiserizing information can be obtained through a different channel involving reputation of the sender who sends public messages through news transmissions even when policies are exogenously given. Also, Chakraborty et al. (2012) report another instance of immiserizing public information where media can affect equilibrium policies in a Hotelling-Downs model with two politicians and an unknown location of the median voter. They show that the media can first affect policy making and then affect electoral outcomes, thereby leading to policy convergence, though the Median Voter Theorem may not hold universally.

The rest of the paper is organized as follows. In Section 2 we describe the model formally. Section 3 studies the case of a single DM. Section 4 deals with media and voting and provides a table that summarizes all our results across different parameter zones. We draw our conclusions in Section 5. Some notation and all proofs are moved to an appendix in Section 6.

2 The Model

An odd number of voters with common preferences vote over two alternatives. Their preferences over the alternatives depend upon an unknown state of the world and each voter receives privately an informative signal about the true state. A media with preferences different from those of the voters knows this state and sends a public message about it. Once all votes cast by the voters are aggregated into a social decision, the voters get to know the true state (as they obtain their payoffs). If the message sent by the media suggests that this true state was im-
possible, the voters punish the media. The media is reputation driven and it cares about this possible punishment. For a media who is not driven by reputation, no information can be transmitted and hence we treat this case as one without a media so that in the rest of the paper, *media is always reputation driven*. We model this environment in the following way.

\[ I = \{1, \ldots, n\} \text{ is the set of voters } (n \geq 1 \text{ and odd}), \quad A = \{X, Y\} \text{ is the set of alternatives and } \Omega = [0, 1] \text{ is the set of states.} \]

*Priors:* The state \( \omega \in \Omega \) is a random variable and agents have a common prior given by density \( f(\omega) \) where \( f \) is non-atomic, and the distribution function is given by \( F(\omega) \). Voters do not observe the true state \( \omega \) while the media observes it.

*Voters’ preference:* Voters have a common preference over \( A \) represented by the state-dependent strict preference relation \( \succ \) such that for some \( 0 < \omega_v < 1 \), we have \( X \succ Y \) if \( \omega \leq \omega_v \) and \( Y \succ X \) if \( \omega > \omega_v \). These preferences of the voters are represented by the utility function \( u : A \times \Omega \to \mathbb{R} \) such that for \( \zeta, \tau \in \mathbb{R}, \zeta < \tau \) we have:

\[
u(X, \omega) =\begin{cases} 
\tau & \text{if } \omega \leq \omega_v \\
\zeta & \text{otherwise}
\end{cases}
\]

and

\[
u(Y, \omega) =\begin{cases} 
\tau & \text{if } \omega > \omega_v \\
\zeta & \text{otherwise}
\end{cases}
\]

*Voters’ private signals:* Each voter \( i \in I \) receives a private signal \( s_i \in \{X, Y\} \equiv S \) whose precision is \( p \in (1/2, 1) \), that is, \( \mathbb{P}[s_i = X|\omega \leq \omega_v] = \mathbb{P}[s_i = Y|\omega > \omega_v] = p \).

*Media’s preference:* The media strictly prefers \( X \) over \( Y \) in all states. This preference of the media is represented by the utility function \( u_m : A \times \Omega \to \mathbb{R} \) such
that for $\zeta_m, \tau_m \in \mathbb{R}$ with $\zeta_m < \tau_m$ we have $u_m(X, \omega) = \tau_m$ and $u_m(Y, \omega) = \zeta_m$ for all $\omega \in \Omega$.

**Size of ex-ante conflict:** The case $F(\omega_v) > 1/2$ will be referred to as a case of small conflict between the voters and the media while large conflict will correspond to $F(\omega_v) < 1/2$.

**Messages:** The media sends a public message (that is observed by all voters). We model messages as follows: For $k \geq 1$, let $\Omega^k = \{\Omega_1, \ldots, \Omega_k\}$ be a $k$-element partition of $\Omega$. Let the message set be $M^k = \{m_1, \ldots, m_k\}$. A message strategy of arity $k$ is a function $m_k : \Omega \rightarrow M^k$ that maps each $\omega \mapsto m_k(\omega) \in M^k$ with the following literal meaning: $m_k(\omega) = m_j$ means $\omega \in \Omega_j$ for all $j = 1, \ldots, k$.

Generic messages will be denoted as $m', m'' \in M^k$. Let $\mathcal{M}$ be the space of message strategies of all arities $k \geq 1$.

**Voting and social decisions:** A voting strategy for voter $i \in I$ is a function $v_i : M^k \times S \rightarrow A$ that maps the received message $m' \in M^k$ and the private signal $s_i$ to generate a vote $v_i \in A$. We denote by $v = (v_1, \ldots, v_n) \in A^n$ a vote profile and use the shorthand $v(m', s)$ to denote $(v_1(m', s_1), \ldots, v_n(m', s_n))$.

The social decision function $\delta : A^n \rightarrow A$ is majoritarian and maps a vote profile $v \in A^n$ to an outcome $\delta(v) \in A$ such that $\delta(v) = X$ if and only if $\#\{i \in I | v_i = X\} \geq \frac{n+1}{2}$.

**Lies and punishments:** Once all the above decisions are taken, the voters observe the true state $\omega$. At this stage, the voters pass a judgment about the ‘trustworthiness’ of the media. Fix any pair $(\Omega^k, M^k)$. Suppose the media uses some arbitrary message strategy $m_k : \Omega \rightarrow M^k$. We call a message $m_j \in M^k, j = 1, \ldots, k$ a lie if $\omega \notin \Omega_j$. In other words, a message that violates (ex-post) the ‘literal meaning’ clause defined above is a lie. The voters dislike lies and punish the media. Let $c$ be the associated cost borne by the respective media in the event it is punished.
for lying. We shall assume $c > \tau_m$.

**Equilibrium**: We focus on symmetric perfect Bayesian equilibria in pure strategies where voters use weakly undominated strategies. Such a voting strategy results in maximum information aggregation (see for example Austen-Smith and Banks (1996)). As well established in the existing literature on strategic voting, a voting equilibrium thus generated is called ‘informative’, and we focus on this equilibrium in what follows. Under this class of equilibria, there is a distinction that is useful for our purposes: one where each vote reveals fully the voter’s private signal (separating equilibrium) and the other where votes reveal no such information (pooling equilibrium). Also, we look at message strategies in equilibria which maximize the media’s ex-ante payoffs, which we call the most influential and report the coarsest among them. By the term equilibrium, in the rest of the analysis we shall mean a coarse, influential and informative equilibrium where voters vote informatively and the media is as influential as permitted by equilibrium conditions. For a formal definition of equilibrium, see Appendix 6.1.

3 Single Decision Maker

We first study the model in a standard sender-receiver setting where there is a single receiver. Moreover, as we shall see in Section 4, much of the analysis we do now will be used directly when we study voting. So, suppose in the model described above, there is a single receiver or decision maker (DM), that is $n = 1$ (we denote this single ‘voter’ by $i$ and use the notation ‘∅’ to denote the absence

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[6] This is similar to Chen (2011). Also see Mertens and Zamir (1985) on conceptualizing mis-information.

[7] In our model this means that each voter follows a voting strategy in which he votes in favor of the alternative that is better for him after having made full use of his available information (which consists of the public message, the private signal received and if possible inference about the signals of the other voters from the pivotal vote profile).

[8] This means that among the set of all ‘most-influential news coverage’, the media uses that which requires minimal partitioning of the state space.
of the media and hence an empty message). We begin with the benchmark case where the media is absent.

**Lemma 1** (Without Media). *Suppose there is a single decision maker, that is \( n = 1 \). His optimal decision \( v_i \) has the following features:

(a) Suppose \( F(w_v) > 1/2 \). Then,

(i) \( v_i(\emptyset, s_i) = s_i \) if \( p > F(\omega_v) \) and

(ii) \( v_i(\emptyset, s_i) = X \) for each \( s_i \in S \) if \( p < F(\omega_v) \);

(b) Suppose \( F(w_v) < 1/2 \). Then,

(i) \( v_i(\emptyset, s_i) = s_i \) if \( p > 1 - F(\omega_v) \) and

(ii) \( v_i(\emptyset, s_i) = Y \) for each \( s_i \in S \) if \( p < 1 - F(\omega_v) \).

If the strength of the private signal is sufficiently weak, then the degree of inference that can be drawn from it is low. In this case when the prior distribution is biased in favor of the alternative \( X \) (that is, \( F(\omega_v) > 1/2 \)), the DM chooses \( X \) irrespective of his private signal. However, if the strength of his private signal is high, then the reliability of the signal prompts the DM to choose according to his private signal. When \( F(\omega_v) < 1/2 \), the probability of states where he prefers \( Y \) is higher than the states where he prefers \( X \). By analogous reasoning, he therefore chooses \( Y \) irrespective of his private signal in case his signal strength is sufficiently low, and chooses according to his signal otherwise.

We next move to the case when the media is present. As it is common knowledge that the media associates a high cost with the subsequent penalization owing to delivery of incorrect information, it can credibly pass any information and this enhances its ability to influence social decisions. The following two lemmas deal with the equilibrium actions in this case. In this framework, news is an interval of the state space. We call a news item *conclusive* if and only if in each feasible state included in the news item, the optimal social decision of the voters remain
the same. Any news that violates this property will be called inconclusive. Also, a news item is called $X$-endorsing ($Y$-endorsing) if the resulting probability mass over the declared interval (before private signals are incorporated) is higher for $X$ ($Y$) than for $Y$ ($X$).

**Lemma 2** (Small Conflict: $F(\omega_v) > 1/2$). Suppose there is a single decision maker with small conflict with the media. Then, there exists a unique $\omega^* > \omega_v$ such that the media announces whether or not $\omega \leq \omega^*$, that is, $\Omega_1 = [0, \omega^*]$ and $\Omega_2 = (\omega^*, 1]$. Moreover,

(a) if $p < F(\omega_v)$, then $\omega^* = 1$ and $v_i(\Omega_1, s_i) = X$ for each $s_i \in S$, i.e., news contains no information, and

(b) if $p > F(\omega_v)$, then $F(\omega^*) = F(\omega_v)/p$ (that is, $\omega^* > \omega_v$), $v_i(\Omega_1, s_i) = X$ for each $s_i \in S$ and $v_i(\Omega_2, s_i) = Y$ for each $s_i \in S$, i.e., $X$-endorsing news is inconclusive while $Y$-endorsing news is fully revealing, and the decision maker always chooses in accordance with the media endorsements.

With small conflict, the prior distribution is biased in favor of the (media’s favorite) alternative $X$. From Lemma 1, it follows that when no information is provided, the DM chooses $X$ irrespective of his private signal if the strength of his private signal is sufficiently low. Since the media prefers the alternative $X$ for all states of the world, this is the ideal scenario for it and therefore it chooses not to transmit any information in equilibrium. However, if the signal strength of the DM is high, under no additional information, he chooses according to his private signal, which prompts the media to intervene in this scenario. The message strategy which the media follows that maximizes its ex-ante payoff is the following: the state space is partitioned into two intervals, one which slants towards $X$ (the left interval $\Omega_1$) while the other towards $Y$ (the right interval $\Omega_2$). If the state belongs to the left interval, then upon receiving the resultant message the DM chooses $X$ irrespective of his private signal. However, the alternative $Y$ is always chosen if the state belongs to the right interval, which is the less preferred alternative of the media.
Under the message strategy that comprises the most influential equilibrium, the length of the $X$-endorsing interval is maximized. To see this, observe that in order to make the DM adopt a pooling strategy of voting $X$, the message should provide credible information that is sufficiently strong in favor of $X$ (which means the mass of states greater than $\omega_v$ that may have generated the same message needs to be sufficiently small) so that the DM chooses $X$ even when he receives a private signal of $Y$. To achieve this end, a single message should be delivered for all states in $[0, \omega_v]$ (which ensures maximal evidence in favor of states for which the voter favors $X$) along with other states in $(\omega_v, 1)$. After having included the entire support $[0, \omega_v]$, the maximum point $\omega^*$ up to which the news can be thought to be generated from a left interval sustaining a resultant pooling strategy of voting $X$ satisfies the condition $F(\omega^*) = F(\omega_v)/p$ which implies that $\omega^* > \omega_v$.

**Remark 1.** Note that $\omega^*$ is a decreasing function of $p$. This implies that the degree of inconclusiveness of news endorsing $X$ goes down as the signal strength rises, that is, a more informed DM receives more information.

We next move to the case where the conflict between the media and the DM is large.

**Lemma 3** (Large Conflict: $F(\omega_v) < 1/2$). Suppose there is a single decision maker with large conflict with the media. Then there exists a unique $\omega^* \in (0, 1)$ where the media announces whether or not $\omega \leq \omega^*$, that is, $\Omega_1 = [0, \omega^*]$ and $\Omega_2 = (\omega^*, 1]$. Moreover,

(a) If $p > 1 - F(\omega_v)$, then

(i) if $F(\omega_v) < 1 - 1/\sqrt{2}$, then $\omega^*$ satisfies $F(\omega^*) = (F(\omega_v)/p) - ((1-p)/p)$, i.e. $\omega^* < \omega_v$, with $v_i(\Omega_1, s_i) = X$ for each $s_i \in S$ and $v_i(\Omega_2, s_i) = s_i$, i.e., $X$-endorsement is fully revealing, $Y$-endorsement is inconclusive, the DM follows only an $X$-endorsement but votes according to private signal with a $Y$-endorsement;
(ii) if \( F(\omega_v) > 1 - 1/\sqrt{2} \), then if \( p < 1/(2(1 - F(\omega_v))) \), we have \( F(\omega^*) = F(\omega_v)/p \) i.e. \( \omega^* > \omega_v \), with \( v_i(\Omega_1, s_i) = X \) for each \( s_i \in S \) and \( v_i(\Omega_2, s_i) = Y \) for each \( s_i \in S \), i.e., \( X \)-endorsements are inconclusive while \( Y \)-endorsements are fully revealing and the DM follows media endorsements; However if \( p > 1/(2(1 - F(\omega_v))) \), then the message strategy and voting behavior is similar to part (a.i).

(b) If \( p < 1 - F(\omega_v) \), then

(i) when \( p < 1/\sqrt{2} \) we have \( F(\omega^*) = F(\omega_v)/p \) i.e. \( \omega^* > \omega_v \), with \( v_i(\Omega_1, s_i) = X \) for each \( s_i \in S \) and \( v_i(\Omega_2, s_i) = Y \) for each \( s_i \in S \), i.e., \( X \)-endorsements are inconclusive while \( Y \)-endorsements are fully revealing and the DM follows media endorsements, and

(ii) when \( p > 1/\sqrt{2} \) we have \( F(\omega^*) = F(\omega_v)/(1 - p) \) i.e. \( \omega^* > \omega_v \), with \( v_i(\Omega_1, s_i) = s_i \) and \( v_i(\Omega_2, s_i) = Y \) for each \( s_i \in S \), i.e., \( X \)-endorsements are inconclusive while \( Y \)-endorsements are fully revealing; however, the DM follows only the \( Y \)-endorsements, but votes according to private signal with an \( X \)-endorsement.

When \( F(\omega_v) < 1/2 \), the prior distribution is biased in favor of the alternative \( Y \) that the media wants to defeat. Consider a highly aware society. If the degree of conflict is very large, the media chooses to follow a bi-partitioned message strategy which induces the DM to follow the media slant if the news is in favor of \( X \), while he votes according to his private signal if the news is in favor of \( Y \). The media finds this voting behavior ex-ante profitable, for it is assured of its most favored alternative \( X \) when \( \omega \in [0, \omega^*) \), while there still is a positive probability that \( X \) will be voted for when \( \omega \in [\omega^*, 1] \). The same message strategy and induced voting behavior is maintained when the degree of conflict is lessened, along with the characteristic that the level of awareness exceeds a threshold value. However, if the awareness level is lower than the threshold limit, the media in a most influential equilibrium follows a two-interval message strategy such that the
DM follows a pooling strategy of choosing $X$ when the state belongs to the left interval endorsing $X$, and $Y$ when it belongs to the right interval endorsing $Y$.

Now consider a sufficiently unaware society where in the media’s absence the DM chooses alternative $Y$ in each state irrespective of his private signal, which is the worst possible outcome for the media. Note that in this case even when the entire length of $[0, \omega_v]$ is included in a single message, the DM cannot be induced to vote for $X$ for the entire region $(\omega_v, 1]$ when he receives a private signal of $X$, because he does not have enough faith in the precision of his signal. In this case, under the most influential equilibrium the media follows a message strategy that satisfies the following characteristics: if the signal strength is greater than a cut-off value, the bi-partitioned message space is such that when the state belongs to the $X$-endorsing interval, the DM chooses according to his private signal. If the state belongs to the $Y$-endorsing interval, the DM chooses alternative $Y$ irrespective of the private signal received. Here the length of the left interval is maximized by including the entire zone of $[0, \omega_v]$ and stretching the support of the interval to the maximal point such that the DM votes according to his private signal for all states in this interval. If the signal strength is less than the cut-off value, then the most influential equilibrium entails a bi-partitioned message space such that the DM chooses $X$ irrespective of his private signal when the state belongs to the $X$-endorsing interval, and $Y$ when it belongs to the $Y$-endorsing interval.

3.1 Welfare

While we have shown that the media can and will influence the decision, it turns out that a single DM weakly benefits from media presence. In this regard, we have the following proposition.

**Proposition 1** (Media is weakly welfare-improving). *In a scenario with a single decision maker, the presence of a media can only enhance ex-ante welfare. In particular,*
(a) Suppose $F(\omega_v) > 1/2$. Then,

(i) When $p < F(\omega_v)$, the decision maker’s welfare is invariant to the presence of the media.

(ii) When $p > F(\omega_v)$, the decision maker’s welfare is higher under the presence of a media;

(b) Suppose $F(\omega_v) < 1/2$. Then,

(i) When $p > 1 - F(\omega_v)$, the decision maker’s welfare is higher under the presence of a media;

(ii) When $p < 1 - F(\omega_v)$, then

- If $p < 1/\sqrt{2}$, the presence of a media is always better for ex-ante voter welfare, whereas
- If $p > 1/\sqrt{2}$, the ex-ante voter welfare is invariant to the presence or absence of the media.

Consider the case of low conflict where solely based on the prior distribution, the inclination of the DM is towards alternative $X$ which the media prefers (that is, when $F(\omega_v) > 1/2$). In this case if the inclination based on prior information exceeds the strength of the private signal of the DM, then no additional information appears in equilibrium as the media’s best alternative always wins with probability 1. Hence in this case the ex-ante welfare of the DM remains same both in the presence or absence of the media. Moreover, in the case where the media finds it optimal to transmit some information through its news, the DM is never worse-off with the news. For example, consider (a.ii) of the proposition. Here by Lemma (2) part (b) it follows that in equilibrium the media makes the DM choose $X$ irrespective of his signal for all states $[0, \omega^*]$, where $\omega_v < \omega^*$. This implies that information provided by the media may over-ride the informative private signal of the DM when $\omega \in (\omega_v, \omega^*)$, prompting him to choose the less desirable alternative for these states. On the other hand, with the media the DM is assured that he will
choose his most favored alternative when $\omega \in [0, \omega_v]$. The net effect is that the ex-ante welfare of the DM is higher with news coverage and hence the presence of a media augments his welfare.

Now consider the case of large conflict, represented by $F(\omega_v) < 1/2$. In this case the power of the media to manipulate the DM is limited due to large conflict of preferences, and hence the information provided by the media can never hurt the ex-ante welfare of the DM.

In other words, we can think of the result in the following manner. There are two sources of information for the single DM: his private signal and the endorsing news provided by the media. Under certain scenarios, news provided can over-ride the private signal of the DM and make him worse-off for some states of the world. However, reliable news also enables him to choose his desired alternative with a higher probability for other states. Since for a single DM, only one informative signal is used to generate final decisions, it can never be that the welfare of the DM is reduced by introducing an additional source of information available from the media, since the potential gains for some states outweigh the potential losses for other states owing to the introduction of this additional information source. In other scenarios, the news provided complements the action of the DM in the following manner: for some states of the world, it induces the DM to vote according to his private signal (which he would would have done anyway even in the absence of the news), while for other states the informational content of the news leads the DM to choose his most preferred alternative without fail. Hence the DM always welcomes the presence of the media. Seidmann (1990) considers information transmission in a single receiver setting with no costly talk where the type of the receiver is private information. In this case, any message provided by the sender induces a distribution of the receiver’s actions across its types, and there may be two messages that induce distributions over actions that are not ordered by stochastic dominance. Different types of senders who agree in their preference rankings of non-stochastic actions may differ in their preference rankings of these
non-ordered distributions, thereby making effective communication possible. However, in our model we focus on costly talk where the preference ranking of every possible type of sender over the distribution of the receiver’s action is identical.

4 Information Transmission and Voting

We have seen that media presence is always welfare improving with a single DM. With multiple decision makers who take decisions via voting, news otherwise useful can coordinate beliefs of voters in such a way that aggregate decisions are less efficient. When the media sends public messages and voters vote strategically, we ask what sort of information is credibly transmitted and when can the media manipulate social decisions in its favor. The results we obtain without a media or a media with small conflict are similar to Lemmas 1 and 2 though the proofs are more involved and provided in the Appendix.

We begin with the following lemma for the game without media.

**Lemma 4** (Voting without Media). *Suppose there are \( n \geq 3 \) independent voters but there is no media presence. The equilibrium voting strategy for each voter \( i = 1, \ldots, n \) is the same as those of a single decision maker described in Lemma 1.*

Lemma 4 is readily understood from the observation that under the simple majoritarian rule, in a separating voting equilibrium where each vote reveals the voter’s private signal, the pivotal vote profile provides perfectly balanced evidence in favor of either alternatives, and hence the decisive piece of private evidence to be considered by the voter is just his own signal as is the case with a single DM. This then leads to the result that the equilibrium behavior of each voter in a multiple voter case (under our equilibrium criterion of informative voting) is the same as the scenario where a single decision is present.

Given Lemma 4, we now allow media presence. We first deal with the case when the conflict of preference between the voters and the media is small.
Lemma 5 (Voting with Small Conflict: $F(\omega_v) > 1/2$). Suppose in the presence of an informed media there are $n \geq 3$ voters and suppose the conflict between the voters and the media is small. Then the equilibrium actions of the media and the voters are same as those with a single decision maker in the presence of the media as described in Lemma 2.

If the inclination of the voter based solely on the prior distribution to vote for $X$ exceeds the strength of his private signal, then no additional information is made available to him even when a media is present, and each voter votes $X$ irrespective of their private signals. In this case the unanimous decision is $X$ for all states of the world which the media prefers. This rationalizes her choice of not transmitting any information to the voters. When the signal strength is high, the intuition behind the particular message strategy followed by the media is identical to the logic provided in explaining Lemma 2.

Apart from news that reveals no information in any state (which appears when the voters’ posteriors sans media news always tilt towards what the media wants), the media’s problem (both in the single DM or multiple voter case) reduces to a choice between two types of informative news coverage: (i) only the $X$-endorsing news is inconclusive and (ii) potentially tri-partitional message strategy where it is possible to have coverage where slants in favor of both $X$ and $Y$-endorsing news is conclusive. When $F(\omega_v) > 1/2$, we prove that type (i) dominates unambiguously both in the single DM or multiple voter case. However, when $F(\omega_v) < 1/2$, the problem gets more nuanced as already reflected in Lemma 3 even with a single DM.

In general, the choice between these two types of news depends crucially on what we call Condition (*). Let

$$J(n, p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j}.$$
**Condition (\(\ast\)):**

\[ J(n, p) \geq \frac{p(2 - F(\omega_v)) - 1}{(2p - 1)(1 - F(\omega_v))} \]

Note that the expression \(J(n, p)\) is the probability that a constituency of size \(n\) and awareness \(p\) makes a correct social decision when voters vote in accordance with their private signals. Then, \((\ast)\) provides a lower bound on this probability. This lower bound increases in \(p\) and decreases in \(\omega_v\).

While \((\ast)\) holds unambiguously when \(F(\omega_v) > 1/2\), it is neither universal nor empty when \(F(\omega_v) < 1/2\). The following lemma specifies equilibrium actions under \(F(\omega_v) < 1/2\) by using \((\ast)\) directly. It shows that only when \(p\) is large, condition \((\ast)\) matters; when it holds, type (i) messaging dominates while when it is violated type (ii) messaging dominates. We then discuss and provide examples concerning the condition. The lemma uses a particular value of \(p\) which we call \(p'\) where

\[ p' = \frac{(27 - 3\sqrt{78})}{6} + \frac{3\sqrt{78} + 27}{6} \approx .76. \]

**Lemma 6** (Voting with Large Conflict: \(F(\omega_v) < 1/2\). Suppose in the presence of an informed media there are \(n \geq 3\) voters and suppose the conflict between the voters and the media is large. Then, there exists a unique \(\omega^* \in (0, 1)\) such that the media announces whether or not \(\omega \leq \omega^*\), that is, \(\Omega_1 = [0, \omega^*]\) and \(\Omega_2 = (\omega^*, 1]\). Moreover,

(a) If \(p > 1 - F(\omega_v)\), then

(i) If \((\ast)\) holds then \(\omega^*\) satisfies \(F(\omega^*) = F(\omega_v)/p\), i.e. \(\omega^* > \omega_v\), with \(v_i(\Omega_1, s_i) = X\) for each \(s_i \in S\) and \(v_i(\Omega_2, s_i) = Y\) for each \(s_i \in S\), i.e., \(X\)-endorsement is inconclusive, \(Y\)-endorsement is fully revealing and voters follow media endorsements;

(ii) If \((\ast)\) does not hold then \(\omega^*\) satisfies \(F(\omega^*) = (F(\omega_v)/p) - ((1 - p)/p)\), i.e. \(\omega^* < \omega_v\), with \(v_i(\Omega_1, s_i) = X\) for each \(s_i \in S\) and \(v_i(\Omega_2, s_i) = s_i\), i.e., \(X\)-endorsement is fully revealing, \(Y\)-endorsement is inconclusive,
voters follow only an X-endorsement but vote according to their private signals with a Y-endorsement;

(b) If $p < 1 - F(\omega_v)$, then

(i) for $n \geq 5$, the message strategy and voting behavior is same as in (a.i).

(ii) for $n = 3$, for all $p < p'$, the message strategy and voting behavior is same as (a.i). When $p > p'$, then $F(\omega^*) = F(\omega_v)/(1 - p)$, i.e. $\omega^* > \omega_v$, with $v_i(\Omega_1, s_i) = s_i$ and $v_i(\Omega_2, s_i) = Y$ for each $s_i \in S$, i.e., X-endorsements are inconclusive while Y-endorsements are fully revealing; however, voters follow only the Y-endorsements but vote according to private signals with an X-endorsement.

From the lemma it follows that the media always transmits news that is slanted either in favor of X or Y. We first discuss the case when the society is sufficiently aware and each voter votes according to his private signal in the absence of the media. Here, if the media is present the kind of news it will choose to transmit depends on whether (*) holds or not.

We now provide some sufficiency conditions for (*) to hold. Let

$$Q(n, p; \omega_v) = J(n, p)(F(\omega_v) - 1)(2p - 1) + p(2 - F(\omega_v)),$$

and note that (*) holds if and only if $Q(n, p; \omega_v) - 1 \leq 0$.

**Highly aware society:** If $p = 1$, then $J(n, p) = 1$. Hence in this case, $Q(n, 1; \omega_v) - 1 = 0$. Also note that $\frac{\partial Q(n, 1; \omega_v) - 1}{\partial p}|_{p=1} = F(\omega_v) > 0$. This shows that when $p \to 1$, the expression $(Q(n, 1; \omega_v) - 1) < 0$. Hence (*) is always satisfied for all $n$ if the precision of the signal received individually by the voters is high enough.

**Large constituency:** Also consider the case when the size of the electorate is
very large. Note that \( J(n, p) \to 1 \) as \( n \to \infty \), and hence corresponding to this case \( Q(n, p; \omega_v) - 1 = p(F(\omega_v) - 1) < 0 \) for all \( p \in (1/2, 1) \). This shows that condition (*) is always satisfied if the number of voters is sufficiently large.

**Intermediate awareness, not-too-large conflict, small constituency:** Moreover, note that \( Q(n, p; \omega_v) - 1 = p(F(\omega_v) - 1) < 0 \) for all \( p \in (1/2, 1) \). This shows that condition (*) is always satisfied if the number of voters is sufficiently large.

**When (*) does not hold:** The complement of (*) is non-empty as well. We construct an example. Let \( n = 3 \) and \( F(\omega_v) = .18 \). In this case for intermediate values of the precision of the private signal (that is when \( .63 < p < .9 \)), (*) is violated while for the cases \( 1/2 < p < .63 \) or \( .9 < p < 1 \), (*) is satisfied. As a specific example, consider \( n = 3 \) and \( p = .7 \). For these values, \( Q(n, p; \omega_v) - 1 \approx .017 > 0 \), and hence (*) is violated in this case.

Given the above discussion, suppose (*) is satisfied. Lemma 6 shows that in this case the media transmits news that is slanted either in favor of \( X \) (in which case the content of the news is inconclusive) or \( Y \) (here the content of the news is fully revealing). In both cases the voters vote in favor of the alternative towards which the news provided is slanted. If, however (*) is violated, the media transmits news that is either slanted conclusively in favor of its own preferred alternative \( X \) or inconclusively in favor of its less preferred alternative \( Y \). Upon receiving news that is decisively slanted towards \( X \), the voters always vote \( X \). However, if they
receive news from the media that is inconclusively slanted towards \( Y \), they vote in accordance to their private signals. This may appear counter intuitive since here the voters support what the media likes when it asks them to do so, but they are careful and use their private information when the media endorses what it does not like. However, it is worth noting that the voters would have voted according to their private signals had the media been absent. Hence, when the media is present, it regulates the information content of the news to make the voters choose for sure the preferred alternative of the media (\( X \)) for as many states as possible, while for others they are left to behave as they would have in the media’s absence. This leaves the media with a positive probability of having the social decision to be \( X \) when \( \omega \in [\omega^*, 1] \).

Now consider a society with low awareness levels wherein in the media’s absence the voters would have been influenced by the prior and unequivocally voted for \( Y \) irrespective of their private signal received. Note that in this case it is impossible for the media to make the voters vote \( X \) for all \( \omega \in (\omega_v, 1] \) when their private signals is \( X \) even when the entire stretch \([0, \omega_v]\) is included in a single message. Hence the voting behavior of part (a.ii) is impossible to replicate here. In this scenario, the media optimally chooses between message strategy in part (a.i) of the lemma versus providing news of the following kind: either the news has an inconclusive slant towards \( X \) that induces the voters to follow their signals, or the news is conclusively slanted towards \( Y \) which the voters follow. It is shown that for a sufficiently large electorate (greater than or equal to five), the former type dominates the latter. This is because under the former, the media is surely able to implement \( X \) when \( \omega \in (\omega_v, \omega^*] \), whereas under the latter type of message strategy \( Y \) will be chosen for these states under a large electorate, since each voter votes according to his private signal.
4.1 Welfare

We now compare ex-ante voter welfare across absence and presence of media. When the prior distribution is biased in favor of $X$, we have the following result.

With low signal precision, the ex-ante welfare of the voter under both cases is the same. Otherwise, the ex-ante welfare of the voter is different for the two cases. Here we may differentiate among the following scenarios: for a sufficiently large size of the electorate (greater than or equal to seven), the ex-ante welfare of the voter is always higher in the absence of the media. We however show that this is not a general feature of the model. If the number of voters is low (either three or five), then for an intermediate range of signal precision, ex-ante welfare of the voter is higher when the media is present, while for extreme values (either high or low) of signal precision ex-ante voter welfare is higher in the absence of the media. These observations are made precise in Proposition 2.

Proposition 2 (Welfare under Small Conflict: $F(\omega_v) > 1/2$). In a scenario with $n \geq 3$ voters, the presence of a media with small conflict may or may not enhance ex-ante welfare. In particular,

(a) When $p < F(\omega_v)$, the ex-ante voter welfare is invariant to the presence of the media.

(b) When $F(\omega_v) < p$,

(i) If $n \geq 7$, the ex-ante voter welfare is higher in the absence of the media;

(ii) If $n < 7$, there exists $1/2 < k^*(n) < p$ such that (1) when $k^*(n) < F(\omega_v) < p$, the ex-ante voter welfare is higher in the absence of the media; (2) when $1/2 < F(\omega_v) < k^*(n)$, there exists $F(\omega_v) < \hat{p}(n, F(\omega_v)) < \hat{p}(n, F(\omega_v)) < 1$ such that for $p \in (F(\omega_v), \hat{p}(n, F(\omega_v))) \cup (\hat{p}(n, F(\omega_v)), 1)$, the ex-ante voter welfare is higher in the absence of the media. However, when $p \in (\hat{p}(n, F(\omega_v)), \hat{p}(n, F(\omega_v)))$, the ex-ante voter welfare is higher when the media is present.
Consider the case when the prior distribution is biased in favor of the alternative $X$. In this case if the signal strength of the voters is sufficiently low, then the media chooses not to transmit any information and the resultant equilibrium (both in the presence and absence of the media) is such that each voter votes $X$ irrespective of his private signal. It therefore follows that the social decision is unanimously chosen to be $X$ and this is invariant to the number of voters. Hence the ex-ante voter welfare is the same whether or not a media is present.

If the signal strength is sufficiently high, then the proposition states that ex-ante voter welfare is higher in the absence of a media for a sufficiently large size of the electorate. The reason is as follows: In the presence of a media, the nature of information provided is such that the voters invariably vote for their less preferred alternative ($X$) when $\omega \in (\omega_v, \omega^*)$. However, they surely vote for their preferred alternative ($X$) when $\omega \in [0, \omega_v]$. Now, consider the scenario where the media is absent, in which case each voter votes according to his own signal. The probability $J(n, p)$ of a correct decision rises as the number of voters increase, since the incidences of informative signals that contribute to the social decision rises. Hence the relative advantage the presence of a media has on voter welfare when $\omega \in [0, \omega_v]$ diminishes as the size of the electorate rises, due to the fact that owing to the large volume of informative private signals aggregated it becomes highly likely that the preferred alternative ($X$) in this range would be the social decision anyway. When $\omega \in (\omega_v, \omega^*)$, (which is the zone of incorrect decision-making under the media), by the same logic it follows that the preferred alternative ($Y$) in this range would be the social decision in the no media case as the size of the constituency rises. Hence the ex-ante voter welfare under no media exceeds that under a media.

Now consider the case when the size of the electorate is sufficiently small. In this case the probability of a correct decision (without media when voters vote according to their private signals) is low, due to the low number of informative signals that are aggregated to form the social decision. We explain the welfare comparison in this case by a numerical example.
Example 1. Suppose $F(\omega_v)$ is a uniform distribution with support $[0, 1]$. Let the number of voters be $n = 3$. It follows that in this case, if $k^*(3) \approx .527 < \omega_v < p$, then the ex-ante voter welfare is always higher under no media. Since the prior distribution is such that for the majority of the states the preferred alternative of the media and the voters coincide, the media uses this to manipulate the voters such that the voters are better-off on their own without the media. The reasoning is completed by the fact that $p > \omega_v$, which means we are commenting on the scenario where the strength of the private signals of the voters is high. Let $1/2 < \omega_v = .51 < .527$. When $p \in (.51, \hat{p}(3) \approx .52197)$ or $p \in (\hat{p}(3) \approx .68804, 1]$, the ex-ante voter welfare is higher in the absence of the media, while for $p \in (.52197, .68804)$, the ex-ante voter welfare is higher under a media.

Note from Lemma 5 that when the voters have a high level of awareness, $\omega^*$ is a decreasing function of $p$. This implies the region over which the media is able to manipulate the voters into voting for their less preferred choice decreases as the strength of their private signal rises. This means when $p$ is low the zone of manipulation $(\omega_v, \omega^*)$ under the media is large, so that the voter welfare is higher in the absence of the media. On the other hand, if $p$ is very high, the probability that the voters will collectively choose the preferred alternative without a media is high for all states, and hence higher voter welfare warrants non-interference from the media. However, for an intermediate range of $p$, the presence of a media (yielding an advantage in the form of a guarantee that the social decision will be the most preferred one for the voters when $\omega \in [0, \omega_v]$) dominates, such that the ex-ante voter welfare is higher under a media relative to the case when the media is absent.

The intermediate range of signal precision for which the presence of the media is desirable for voter welfare shrinks as the size of the electorate rises from three to five. The logic behind the result is the following: the ex-ante voter welfare without the media increases as the size of the electorate goes up, while the ex-ante voter welfare under media presence is invariant to the number of voters. Hence the zone
where the absence of media is relatively more advantageous for voter welfare goes up as the size of the electorate rises, which alternatively implies that the zone where the presence of the media is preferable for achieving higher voter welfare shrinks.

The following proposition deals with the last case to be considered where the prior distribution is biased in favor of $Y$. There may still arise cases when additional information available from the media may hurt voter welfare, while in others additional information augments voter welfare.

**Proposition 3** (Welfare under Large Conflict: $F(\omega_v) < 1/2$). In a scenario with $n \geq 3$ voters, the presence of a media with large conflict may or may not enhance ex-ante welfare. In particular,

(a) Let $1 - p < F(\omega_v)$. If (*) is satisfied, then ex-ante voter welfare is higher in the presence of the media iff $F(\omega_v) < \frac{p}{1-p}(1 - J(n,p))$. If (*) is violated, then the ex-ante voter welfare is always higher in the presence of the media.

(b) Let $F(\omega_v) < 1 - p$. In this case the presence of the media always corresponds to higher ex-ante welfare for the voter.

Consider the case of a sufficiently aware society where each voter votes according to his private signal in the absence of the media. As discussed before, (*) is always satisfied for all $p \in (\frac{1}{2}, 1)$ when $n \to \infty$, in which case we have $J(n,p) = 1$. Since $F(\omega_v) > 0$, it follows that $F(\omega_v) > \frac{p}{1-p}(1 - J(n,p))$ and thus if the size of the electorate is very large, ex-ante voter welfare is higher in the absence of the media. This is because of the following: in the media’s presence the probability of a correct decision is invariant to the number of voters since each of them follows a signal invariant (or pooling) voting strategy where the decision is always unanimous. In its absence the voters vote according to their private signals which implies that the large number of private signals aggregated to form the social decision guarantees that the correct social decision will be arrived at with a very high
probability. Thus for a very large electorate, the presence of a manipulative media hurts welfare.

If however, the size of the electorate is low, then whether the presence of the media leads to higher ex-ante welfare or not depends on the ‘relative size’ of the set of states for which decision-making is improved owing to the information provided by the media vis a vis the set of states over which the voters are manipulated to vote for their less favored alternative. This in turn is compared to the scenario where the media is absent. Both the cases where media presence or absence is desirable for higher ex-ante voter welfare is feasible, which we demonstrate by citing two examples where the number of voters is low and (*) holds.

**Example 2** (Condition (*) holds, media presence improves welfare.). For the first example, let $F(\omega_v) = .35$, $p = .66$ and $n = 3$. It is already shown after Lemma 6 that (*) is satisfied for these values of the parameters. For these values, $\frac{p}{1-p}(1 - J(n,p)) \approx .521 > .35 = F(\omega_v)$, and hence in this case the presence of the media leads to higher ex-ante voter welfare.

**Example 3** (Condition (*) holds, media presence hurts welfare.). Let $F(\omega_v) = .48$, $p = .74$ and $n = 3$. In this case the conditions $\frac{1}{3} < F(\omega_v) < \frac{1}{2}$ and $1 - F(\omega_v) < p \leq \frac{1+F(\omega_v)}{2}$ holds, and hence (*) is valid which is checked by noting that corresponding to these values, $(Q(n,1;\omega_v) - 1) \approx -.083 < 0$. However, in this case $\frac{p}{1-p}(1 - J(n,p)) \approx .477 < .48 = F(\omega_v)$, and hence in this case the absence of the media leads to higher ex-ante voter welfare.

Now consider the case where (*) is violated, an example of which has been provided after Lemma 6. In this case, due to transmission of news by the media, the voters follow a signal invariant strategy of voting $X$ for a certain range of states contained entirely in $[0, \omega_v]$, which is their preferred alternative for these states of the world. For the rest of the states the voters vote according to their private signals, which they would have done anyway even in the absence of the media. Hence ex-ante voter welfare is higher in the presence of the media when (*)
is violated. Now consider the case where the precision of signals of the voters is sufficiently low such that in the media’s absence the voters vote for $Y$ irrespective of their signal received. Since the states where the preferred alternative of the media and the voters are different are more likely to occur given the prior distribution, the power of the media to manipulate the voters is limited, and given this the voter welfare is higher if information from the media is received. A summary of all results of this section can be found in Table 1.

We end this section by discussing some other aspects and implications of this model.

### 4.2 Voting versus Delegation

In a cheap talk setting with a single receiver, Dessein (2002) shows that full delegation of decision making rights to the informed sender is better than cheap talk communication if the degree of conflict is not large. In a similar setting Ivanov (2010) studies the case where it is now possible to limit the degree of precision of the expert’s information, but not the content. The paper studies the welfare effects of information transmission versus directly delegating the expert to take the decision to show that it may not be in the best interest of the principal to delegate authority to the most informed subordinate.

In relation to these two works, an important feature of our multiple-receiver model is that under no circumstances can it be better for the voters to give up their rights to vote and delegate decision making authorities to the media or the informed elite. To see this first note that if the media becomes the DM, then the social decision is $X$ and the expected payoff of each voter is simply $F(\omega_v)\tau + (1 - F(\omega_v))\zeta$. Suppose first that $F(\omega_v) > 1/2$. If $p < F(\omega_v)$ then we have shown that the outcome from voting is $X$ independent of whether there is a reputation driven media or not. Hence, in this case delegation to the media cannot help the voters. If $p > F(\omega_v)$, then the payoff of the voter in the absence of a media is simply $J(n,p)\tau + (1 - J(n,p))\zeta$ which is higher than $F(\omega_v)\tau + (1 - F(\omega_v))\zeta$, the voter's
<table>
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<tr>
<th>Media Activity and Influence</th>
<th>Small Conflict: $F(\omega_p) &gt; 1/2$</th>
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<tr>
<td><strong>Unaware Society</strong></td>
<td>$p &lt; F(\omega_p)$</td>
<td>$p &lt; 1 - F(\omega_p)$</td>
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<tr>
<td><strong>Aware Society</strong></td>
<td>$p &gt; F(\omega_p)$</td>
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<td><strong>Nature of News</strong></td>
<td>Babble</td>
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<td></td>
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<td><strong>Impact on Voting</strong></td>
<td>Each voter votes for $X$</td>
<td>Voters always vote for media slants</td>
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<tr>
<td></td>
<td>Each voter votes for $X$</td>
<td>(a) If $n &gt; 3$ or if $n = 3, p &lt; 0.76$, then voters always vote for media slants</td>
</tr>
<tr>
<td></td>
<td>Each voter votes for $X$</td>
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<td></td>
<td>Each voter votes for $X$</td>
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<td><strong>Impact on Welfare</strong></td>
<td>No impact</td>
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<td></td>
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<td></td>
<td>No impact</td>
<td>(a) If (*) holds: Media presence can hurt</td>
</tr>
<tr>
<td></td>
<td>No impact</td>
<td>(b) If (*) does not hold: Media always helps.</td>
</tr>
</tbody>
</table>

Table 1: A summary of all results with Voting
payoff from delegation, if and only if $J(n, p) > F(\omega_v)$. But for any $n > 1$ and any $p > 1/2$ we have $J(n, p) > p$, and since $p > F(\omega_v)$ it follows that for any $n \geq 3$ and any $p > 1/2$ we have $J(n, p) > F(\omega_v)$. Hence delegation cannot be first best. Now suppose $F(\omega_v) < 1/2$. In this case when $p > 1 - F(w_v)$, then in the absence of a media the voting equilibrium is separating. In this case, democracy beats delegation if $J(n, p) > F(w_v)$, which always holds since $J(n, p) > 1/2 > F(w_v)$. When $p < 1 - F(w_v)$, then in the absence of a media the the voting equilibrium is pooling (on alternative $Y$) so that voter’s payoff is given by $F(w_v)\zeta + (1 - F(w_v))\tau$ while under delegation it is given by $F(w_v)\tau + (1 - F(w_v))\zeta$. Since $F(w_v) < 1/2$, the welfare with no media beats welfare under delegation. Hence, in no circumstance can the society be better off by delegating decision making authorities to the media.

4.3 On regulating the media

We have established that media presence can hurt. This result suggests interesting policy implications. To address this, it is more convenient to re-interpret the model slightly. Suppose for the moment that the voters can neither avoid the media nor directly punish it for transmitting misleading news. However, there is a regulatory authority that potentially can penalize the media. If it decides to impose a fine (like cancellation of telecast rights) for misleading news, and if this is common knowledge, then the model studied above can be readily used to ask what the authority should do.

Observe that if the media is not regulated in this sense, it resembles the case when no information can be transmitted credibly (that is, absence of a reputed media). Our results suggest in certain circumstances (characterized by $p$, $F$ and $n$), the optimal policy is then either to ban the media outright (so that it cannot transmit any news) or deregulate it completely (so that it cannot credibly transmit any information even if it tried). Since banning the media is most of the time politically infeasible, deregulation is the only option. By deregulating, the authority takes away credibility of the media and our results show this can be beneficial.
Of course, our results also suggest that under certain other circumstances when media presence helps the voters, the authorities should strengthen regulatory punishments.

4.4 Reputation-building and endogenous viewership

Fix viewership first to $n$. If the media is aware that by gaining in reputation it can improve its own payoffs, it is fair to say that some sort of truthful reporting should appear in a repeated setting. By subsequently reporting truthful news, the media builds the credibility of future news. Interestingly, under certain values of $p$ and $F$ the ex-ante welfare of the voters in each period is lower than the case where the media was not engaged in an effort to build a reputation through truthful reporting.

Further, if media’s presence can hurt the voters, can the voter choose to stay away from media news? Durante and Knight (2012) examines whether and how viewers in Italy respond to changes in partisan bias in media news. They find robust evidence that viewers responded to these changes by modifying their choice of favorite news programs. So consider the scenario where the viewership of news is decentralized in the sense that each of the voters endogenously choose whether to access the news transmitted by the media. In our setup, full participation in the acquisition of news is the unique Nash equilibrium, which then becomes equivalent to the model we study. This is because ceteris paribus, as our analysis of a single DM reveals, more information always increases that voter’s expected payoff. This coupled with the fact that in a multi-player scenario each voter votes as-if-pivotal would mean that each voter will always be willing to improve his information prior to voting.
5 Conclusion

In this paper we study the effect of information transmission by a perfectly in-
formed and partially biased media to less informed voters. We show that the
presence of media never hurts welfare if there is a single decision maker or when
multiple voters are present with low awareness levels. If the voters are sufficiently
aware, then media presence hurts welfare for large electorates. The strategically
chosen content of the informative news transmitted by the media overpowers the
private information of the voters and invariably makes them vote for a particular
alternative. In contrast, without the media the voters would have voted according
to their private signals so that the probability of the correct decision increases with
the size of the constituency. Hence media absence can improve welfare in large
constituencies. This perverse effect of media presence can also appear in small
constituencies, though not universally.

One may also investigate how the results are altered if we consider a multiple
alternatives voting model and/or apply other aggregation rules such as the system
of approval voting or cumulative voting. It is also quite natural to introduce
multiple media outlets with either like or conflicting biases (in a set-up similar
to Krishna and Morgan (2001), Gilligan and Krehbiel (1989)) who may either
sequentially or simultaneously deliver messages to the voters, and examine their
implication on public welfare. We reserve this for future research. Needless to
mention, our work is not to suggest that media is not useful. As our results show,
in many instances they are.

6 Appendix

6.1 Notations and formal definitions

Fix the arity of the message strategy to $k$. A perfect Bayesian equilibrium (PBE)
is a strategy profile $(m_k, v)$ such that:
(i) for each \( s_i \in \{X,Y\} \), and \( i \in I \) we have

\[
\sum_{s_{-i} \in \{X,Y\}^{n-1}} \left( \int_{\omega \in \Omega_k} \mathbb{P}[s_{-i} \mid m_k(\omega), s_i] u(\delta(m_k(\omega), (s_i, s_{-i})), \omega) f(\omega \mid s) d\omega \right) \geq \\
\sum_{s_{-i} \in \{X,Y\}^{n-1}} \left( \int_{\omega \in \Omega_k} \mathbb{P}[s_{-i} \mid m_k(\omega), s_i] u(\delta(m_k(\omega), (s_i, s_{-i})), v_{-i}, \omega) f(\omega \mid s) d\omega \right),
\]

and

(ii) for each realized state \( \omega \in \Omega \) and each \( \omega' \notin \Omega(m_k(\omega)) \), we have

\[
\sum_{s \in \{X,Y\}^n} \mathbb{P}[s \mid \omega] u_m(\delta(v(m_k(\omega)), s), \omega) \geq \sum_{s \in \{X,Y\}^n} \mathbb{P}[s \mid \omega] u_m(\delta(v(m_k(\omega')), s)), \omega) - c.
\]

Given two equilibria \((m_k, v)\) and \((m_{k'}, v')\), we say that they are decision-equivalent if for every \( \omega \in \Omega \), \( v(m_k, s) = v'(m_{k'}, s) \). We say \((m_k, v)\) is decision-equivalent coarsening (DEC) of \((m_{k'}, v')\) if they are decision equivalent and \( k < k' \).

We shall always work with maximal DECs, that is, given the set of all decision equivalent equilibria, we shall only consider those which are the coarsest in this set.

In the voting sub-game an equilibrium is pooling when \( v_i(m_k(\omega), s_i) = v_j(m_k(\omega), s_j) \) for any \( i, j \in I \) for any \( s_i, s_j \in \{X,Y\} \). Similarly an equilibrium is separating when \( v_i(m_k(\omega), X) \neq v_i(m_k(\omega), Y) \) for any \( i \in I \).

Let \( \Xi \) be the set of all possible informative equilibria. An informative equilibrium \((m^*_k, v^*) \in \Xi\) is called most influential if for all equilibria \((m_k, v) \in \Xi\), we have

\[
\int_{\omega \in \Omega} \left( \sum_{s \in \{X,Y\}^n} \mathbb{P}[s \mid \omega] u_m(\delta(v^*(m^*_k(\omega)), s), \omega) \right) f(\omega) d\omega \\
\geq \int_{\omega \in \Omega} \left( \sum_{s \in \{X,Y\}^n} \mathbb{P}[s \mid \omega] u_m(\delta(v(m_k(\omega)), s), \omega) \right) f(\omega) d\omega.
\]

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We identify social welfare in terms of the \textit{ex-ante welfare} of the voters prior to any non-prior information received. Note that this is fully explained by any individual voter’s preferences since all voters are ex-ante identical and each represents the preferences of the public. Let $U(m_k, v)$ be the \textit{ex-ante welfare of a single voter} under a strategy profile $(m_k, v)$. Then

$$U(m_k, v) = \int_{\omega \in \Omega} \left( \sum_{s \in \{X, Y\}} P[s|\omega] u(v(m_k(\omega), s), \omega) \right) f(\omega) d\omega.$$ 

### 6.2 Proofs

**Proof of Lemma 1**

Let $\gamma_{s_i} = P[\omega \leq \omega_i|s_i]$. In the absence of the media, let the expected utility of the voter $i$ from voting $v_i$ when he receives a private signal of $s_i$ be denoted by $U_{v_i}(\emptyset, s_i)$. The voter $i$ votes for $X$ if and only if $U_X(\emptyset, s_i) > U_Y(\emptyset, s_i)$, from which it follows that $(2\gamma_{s_i} - 1)\tau > (2\gamma_{s_i} - 1)\zeta$. Since $\tau > \zeta$, the above inequality holds if and only if $(2\gamma_{s_i} - 1) > 0$, which implies $\gamma_{s_i} > 1/2$. Hence it follows that $v_i(\emptyset, X) = X$ if $\gamma_X > 1/2$ and $v_i(\emptyset, X) = Y$ if $\gamma_X < 1/2$. Similarly, $v_i(\emptyset, Y) = X$ if $\gamma_Y > 1/2$ and $v_i(\emptyset, Y) = Y$ if $\gamma_Y < 1/2$.

Consider $F(\omega_v) > 1/2$. In this case the condition $\gamma_X > 1/2$ implies $p > 1 - F(\omega_v)$, which always holds since $p > 1/2$. The condition $\gamma_Y < 1/2$ implies $p > F(\omega_v)$. The binding condition for $v_i(\emptyset, s_i) = X$ for each $s_i \in S$ is therefore $p > F(\omega_v)$. This proves part (a.i) of the lemma. Note that when $p < F(\omega_v)$, then $\gamma_X > 1/2$ and $\gamma_Y > 1/2$. This proves part (a.ii) of the lemma.

Now consider $0 < F(\omega_v) < 1/2$. In this case the condition the condition $\gamma_Y < 1/2$ implies $p > F(\omega_v)$, which always holds since $F(\omega_v) < 1/2 < p$. The condition $\gamma_X > 1/2$ implies $p > 1 - F(\omega_v)$, which is therefore the binding condition for $v_i(\emptyset, s_i) = s_i$ for each $s_i \in S$. This proves part (b.i) of the lemma. Note that when $p < 1 - F(\omega_v)$, then $\gamma_Y < 1/2$ and $\gamma_X < 1/2$ holds. This proves part (b.ii) of the lemma and concludes the proof.
Proof of Lemma 2:
When $1/2 < p < F(\omega_v)$, from Lemma 1 part (a.ii) it follows that in the voting subgame of the informative equilibrium the voting strategy $v_i$ is such that the preferred alternative of the media is implemented for all states $\omega \in [0, 1]$, and hence there does not exist any profitable deviation for the media from $\omega^* = 1$.
This proves part (a) of the lemma.

Let $1/2 < F(\omega_v) < p$. Consider the equilibria under following class of message strategy with arity $k = 2$, which we classify as Class 1, under which $\Omega_1 = [0, \hat{\omega})$, $\Omega_2 = [\hat{\omega}, 1]$, where $\omega_v < \hat{\omega}$ such that $v_i(m_1, s_i) = X$ and $v_i(m_2, s_i) = Y$ for each $s_i \in S$.
Suppose the message provided is $m_1$. In this case voter $i$ considers the following posterior probability given by
\[
\gamma'_{s_i} = P[\omega < \omega_v|s_i, m_1] = \frac{G}{H},
\]
where
\[
G = P[s_i, m_1|\omega < \omega_v]P[\omega < \omega_v]
\]
and
\[
H = P[s_i, m_1|\omega < \omega_v]P[\omega < \omega_v] + P[s_i, m_1|\omega_v < \omega < \hat{\omega}]P[\omega_v < \omega < \hat{\omega}]
\]
Voter $i$ votes $v_i(m_1, s_i) = X$ iff $\gamma'_{s_i} > 1/2$, and $v_i(m_1, s_i) = Y$ otherwise. We have,
\[
\gamma'_{X} = \frac{pF(\omega_v)}{pF(\omega_v) + (1 - p)(F(\hat{\omega}) - F(\omega_v))}
\]

and
\[
\gamma'_{Y} \frac{(1 - p)F(\omega_v)}{(1 - p)F(\omega_v) + p(F(\hat{\omega}) - F(\omega_v))}.
\]
Note that \(\gamma'_{Y} = 1/2\) when \(F(\hat{\omega}) = \frac{F(\omega_v)}{p}\). Since \(\frac{\partial \gamma'_{Y}}{\partial F(\hat{\omega})} < 0\), it follows that when \(F(\hat{\omega})\) is greater (lesser) than \(\frac{F(\omega_v)}{p}\), then \(\gamma'_{Y}\) is lesser (greater) than \(1/2\). Since \(p > 1/2\), we have \(\gamma'_{s_i = X} > \gamma'_{s_i = Y}\). Therefore when \(\Omega_1 = [0, \hat{\omega}]\) where \(F(\hat{\omega}) = \frac{F(\omega_v)}{p}\) is satisfied, the voter votes \(v_i = X\) for each \(s_i \in S\) in a symmetric informative equilibrium. For the case when \(m_2 = (\hat{\omega}, 1]\), it follows that \(P[\omega \leq \omega_v|m_2, s_i] = 0\) for each \(s_i \in S\), and hence in an informative equilibrium the voter \(i\) votes \(v_i = Y\) for each \(s_i \in S\). This proves that \((m_k, v)\) is an equilibrium.

Note that in this case the ex-ante payoff of the media under Class 1 message strategy is given by
\[
\mathbb{E}[u_m]_1 = \int_0^{\hat{\omega}} \tau_m f(\omega)d\omega + \int_{\hat{\omega}}^1 \zeta_m f(\omega)d\omega
\]
which simplifies as
\[
\mathbb{E}[u_m]_1 = F(\hat{\omega})(\tau_m - \zeta_m) + \zeta_m
\]
Since \(\tau_m > \zeta_m\), it follows that \(\frac{\partial \mathbb{E}[u_m]_1}{\partial \hat{\omega}} > 0\), which implies that by choosing \(\hat{\omega}\) that satisfies the condition \(F(\hat{\omega}) = \frac{F(\omega_v)}{p}\), the ex-ante payoff of the media is maximized under Class 1 message strategies. Hence the optimum value of ex-ante payoff of the media under Class 1 message strategies is given by
\[
\mathbb{E}[u_m]_1^* = \frac{F(\omega_v)}{p}(\tau_m - \zeta_m) + \zeta_m
\]
Consider the equilibria in the following class of message strategies, defined as Class 2.

Class 2: Suppose there exists \(\alpha \in [0, \omega_v)\), \(\beta \in (\omega_v, 1]\) such that the message
strategy function \( m_k \) followed by the reputation driven media is such that \( \Omega_1 = [0, \alpha), \Omega_2 = [\alpha, \beta), \Omega_3 = [\beta, 1] \), where the voting strategy followed is:

\[
v_i = \begin{cases} 
X & \text{if } \Omega_1 = [0, \alpha) \\
s_i & \text{if } \Omega_2 = [\alpha, \beta] \\
Y & \text{otherwise}
\end{cases}
\]

Note that when \( m_2 = (\alpha, \beta] \) is received, then in an informative and symmetric equilibrium for \( v_i = s_i \) for each \( s_i \in S \) to hold, the conditions

\[
\hat{\gamma}_X = \mathbb{P}[\omega \leq \omega_v | s_i = X, m_2 = (\alpha, \beta)] > 1/2
\]

and

\[
\hat{\gamma}_Y = \mathbb{P}[\omega \leq \omega_v | s_i = Y, m_2 = (\alpha, \beta)] < \frac{1}{2}
\]

need to be satisfied simultaneously. The first inequality reduces to

\[
\frac{p}{1-p}(F(\omega_v) - F(\alpha)) + F(\omega_v) > F(\beta)
\]

while the second becomes

\[
\frac{1-p}{p}(F(\omega_v) - F(\alpha)) + F(\omega_v) < F(\beta).
\]

Under this class the ex-ante payoff of the media is denoted by \( \mathbb{E}[u_m]_2 \) where

\[
\mathbb{E}[u_m]_2 = \int_0^\alpha \tau_m f(\omega) d\omega + \int_\alpha^{\omega_v} (p \tau_m + (1-p)\zeta_m) f(\omega) d\omega + \\
\int_{\omega_v}^\beta ((1-p) \tau_m + p \zeta_m) f(\omega) d\omega + \int_\beta^1 \zeta_m f(\omega) d\omega
\]
which is simplified as

$$\mathbb{E}[u_m]_2 = (F(\alpha) + F(\beta))(1 - p)(\tau_m - \zeta_m) + F(\omega_v)(\tau_m - \zeta_m)(2p - 1) + \zeta_m \quad (2)$$

Since $\tau_m > \zeta_m$ and $1/2 < p < 1$, from (2) it follows that in order to maximize the ex-ante payoff of the media, the following maximization problem needs to be solved, which we denote as $(\ast)$.

Maximize $(F(\alpha) + F(\beta))$ subject to: (i) $\frac{p}{1-p}(F(\omega_v) - F(\alpha)) + F(\omega_v) > F(\beta)$,

(ii) $\frac{1-p}{p}(F(\omega_v) - F(\alpha)) + F(\omega_v) < F(\beta)$, (iii) $0 \geq F(\alpha) < F(\omega_v)$, and (iv) $F(\omega_v) < F(\beta) \leq 1$.

Setting $F(\alpha) = 0$ and considering equality in constraint (i), we have $F(\beta) = \frac{F(\omega_v)}{1-p}$. Note that since $p > 1/2$ and $F(\omega_v) > 1/2$, therefore the condition $1 \geq \frac{F(\omega_v)}{1-p}$ can never hold. Now consider $1 < \frac{F(\omega_v)}{1-p}$. In this case from the optimization problem $(\ast)$ it follows that the optimum value is given by the relation $F(\beta^*) = 1$, which implies $\beta^* = 1$. Putting the optimum value of $F(\beta^*)$ in constraint (i), we have $F(\alpha^*) = \frac{F(\omega_v)}{p} - \frac{1-p}{p}$. Replacing the optimum values of $F(\alpha^*)$ and $F(\beta^*)$ in equation (2), we have the maximum ex-ante payoff of the media under equilibria belonging to Class 2 to be

$$\mathbb{E}[u_m]_2^* = \left( \frac{F(\omega_v)}{p} - \frac{1-p}{p} + 1 \right) (1-p)(\tau_m - \zeta_m) + F(\omega_v)(\tau_m - \zeta_m)(2p-1) + \zeta_m \quad (3)$$

After having described the equilibria which corresponds to the highest ex-ante payoff of the media in Class 1 and Class 2 for the single receiver case, we compare the ex-ante payoff of the media between these two equilibria. From equations (1) and (3) it follows that

$$\mathbb{E}[u_m]_2^* - \mathbb{E}[u_m]_1^* = \frac{E}{p} \quad (4)$$

where $E = (\tau_m - \zeta_m)(2p^2(F(\omega_v) - 1) + p(3 - 2F(\omega_v))) - 1$
Since $\tau_m > \zeta_m$, we have $E[u_m]^* - E[u_m]^* < 0$ if
\begin{equation}
2p^2(F(\omega_v) - 1) + p(3 - 2F(\omega_v)) - 1 < 0 \tag{5}
\end{equation}

Let $D(p, F(\omega_v)) = 2p^2(F(\omega_v) - 1) + p(3 - 2F(\omega_v)) - 1$. Note that $\frac{\partial D(p, F(\omega_v))}{\partial F(\omega_v)} = 2p(p - 1) < 0$. Therefore the maximum value of $D(p, F(\omega_v))$ is attained at $D(p, 1/2) = -(p - 1)^2 < 0$. Hence $D(p, F(\omega_v)) < 0$ for all $p \in (1/2, 1), F(\omega_v) \in (1/2, 1)$. This proves that for the single voter case, the optimal equilibrium obtained under Class 1 corresponds to a higher level of ex-ante payoff for the media than the optimal equilibrium obtained under Class 2.

We now argue that the optimum equilibrium obtained under Class 1 corresponds to the maximum ex-ante payoff of the media among all possible equilibria. Note that under Class 1 equilibria, $E[u_m]_1$ is increasing in $\hat{w}$, and the maximum value of $\hat{w}$ under Class 1 is when the condition $F(\hat{w}) = F(\omega_v)p$ is satisfied. When $F(\hat{w}) > F(\omega_v)p$ then $\gamma'_Y < 1/2$ and we revert to equilibria under Class 2 with $\alpha = 0$, $\beta = \hat{w}$ which has been proved to have a lower expected payoff for the media than the optimum Class 1 equilibrium. This proves part (b) and concludes the proof.

**Proof of Lemma 3**: 
Now consider the case when a media is present when $F(\omega_v) < 1/2$.

Analogous to the proof of Lemma 2, it follows that the most influential equilibria under Class 1 type of message strategies is achieved by choosing $\hat{\omega}$ such that the condition $F(\hat{\omega}) = \frac{F(\omega_v)}{p}$ is satisfied. Under this class the expected utility of the media is given by equation (1).

Now we consider Class 2 type of message strategies which gives rise to optimization problem (*) as defined in the proof of Lemma 2 part (b). Let $p > 1 - F(\omega_v)$, which implies $F(\omega_v) > 1 - p$. In this case, analogous to the proof of Lemma 2, it follows that $\beta^*$ and $\alpha^*$ are such that $F(\beta^*) = 1$ and $F(\alpha^*) = \frac{F(\omega_v)}{p} - \frac{1-p}{p}$. Hence the expression for $E[u_m]^* - E[u_m]_2$ for this case is given by (4). Consider the ex-
pression $D(p, F(\omega_v))$ as defined in the proof of Lemma 2. From (4) it follows that $\mathbb{E}[u_m]_2^*$ is greater (lesser) than $\mathbb{E}[u_m]_1^*$ if $D(p, F(\omega_v))$ is positive (negative). Note that $D(1, F(\omega_v)) = 0$ and $\frac{\partial D(p, F(\omega_v))}{\partial p} |_{p=1} = 2F(\omega_v) - 1 < 0$, since $F(\omega_v) < 1/2$. Also note that $D(1/2, F(\omega_v)) = -\frac{F(\omega_v)}{2} < 0$ and $\frac{\partial D(p, F(\omega_v))}{\partial p} |_{p=1/2} = 1 > 0$. The expression $D(p, F(\omega_v)) = 0$ has a unique solution in $p \in (1/2, 1)$ that is given by $\bar{p} = \frac{1}{2(1-F(\omega_v))}$. This proves that for all $p$ lesser (greater) than $\bar{p}$, the expression $D(p, F(\omega_v))$ is negative (positive). Now, if the condition $1 - F(\omega_v) < p < \bar{p} = \frac{1}{2(1-F(\omega_v))}$ has to hold, we must have $1 - F(\omega_v) < \frac{1}{2(1-F(\omega_v))}$ which holds iff $F(\omega_v) > \left(1 - \frac{1}{\sqrt{2}}\right) \approx .29$. Hence if $F(\omega_v) \leq .29$, then $\frac{1}{2(1-F(\omega_v))} \leq 1 - F(\omega_v) < p$ and $D(p, F(\omega_v)) > 0$ always holds. This proves part (a.i). However, if $F(\omega_v) > .29$, then if $1 - F(\omega_v) < p < \bar{p}$, we have $D(p, F(\omega_v)) < 0$ but if $1 - F(\omega_v) < \bar{p} < p$, then $D(p, F(\omega_v)) > 0$. This proves part (a.ii) of the lemma.

Now consider $p < 1 - F(\omega_v)$. In this case the solution of optimization problem (4) is given by $\alpha^*$ and $\beta^*$ such that $F(\alpha^*) = 0$ and $F(\beta^*) = F(\omega_v) = \frac{F(\omega_v)}{1-p}$. Putting $\alpha^*$ and $\beta^*$ in equation (2) we have the ex-ante payoff of the media corresponding to the most influential message strategy under Class 2 to be

$$\mathbb{E}[u_m]_2^* = \left(\frac{F(\omega_v)}{1-p}\right)(1-p)(\tau_m - \zeta_m) + F(\omega_v)(\tau_m - \zeta_m)(2p - 1) + \zeta_m \quad (6)$$

From equations (1) and (6) we have

$$\mathbb{E}[u_m]_2^* - \mathbb{E}[u_m]_1^* = \frac{F(\omega_v)(\tau_m - \zeta_m)(2p^2 - 1)}{p}$$

Since $F(\omega_v) > 0$, $\tau_m > \zeta_m$, it follows that $\mathbb{E}[u_m]_2^*$ is lesser (greater) than $\mathbb{E}[u_m]_1^*$ if $p$ is lesser (greater) than $\frac{1}{\sqrt{2}}$. This proves parts (b.i) and (b.ii) of the lemma and concludes the proof.

Proof of Proposition 1:

When $1/2 < p < F(\omega_v)$, from Lemma 1 case (a.ii) and Lemma 2 case (a) it follows
that under no media or with media the ex-ante welfare of the DM is

\[ U(\emptyset, v) = F(\omega_v)\tau + (1 - F(\omega_v))\zeta \]

This proves part (a.i) of the proposition.

When \( \frac{1}{2} < F(\omega_v) < p \), then from Lemma 1 part (a.i) it follows that

\[ U(\emptyset, v) = p\tau + (1 - p)\zeta \] \hspace{1cm} (7)

In the presence of the media it follows from Lemma 2 part (b) that in the most influential informative equilibrium, the ex-ante welfare of the DM is

\[ U(m_k, v) = \tau + F(\omega_v)(\tau - \zeta) \left( \frac{p - 1}{p} \right) \] \hspace{1cm} (8)

From (7) and (8) it follows that \( U(m_k, v) - U(\emptyset, v) > 0 \) iff

\[ (p - 1)(F(\omega_v) - p) > 0 \] \hspace{1cm} (9)

which always holds since \( F(\omega_v) < p < 1 \). This proves part (a.ii) of the proposition.

Now consider \( F(\omega_v) < \frac{1}{2} \).

When \( p > 1 - F(\omega_v) \), then from Lemma 1 part (b.i) it follows that in the absence of the media, the ex-ante voter welfare is given by (7). Now consider presence of the media. In the instances when the optimal Class 1 message is delivered, the ex-ante welfare of the voter is given by (8). Since \( F(\omega_v) < \frac{1}{2} < p < 1 \), therefore (9) always holds, and hence \( U(m_k, v) - U(\emptyset, v) > 0 \) in this case. In the instances when optimal Class 2 message is provided by the media, it is easy to check that \( U(m_k, v) - U(\emptyset, v) > 0 \), since in the absence of the media the DM always votes according to his private signal, while in its presence he votes according to his signal for \( \omega \in [\omega^*, 1] \), while being guaranteed of his most preferred alternative \( X \) when \( \omega \in [0, \omega^*) \). This proves part (b.i) of the proposition.
Suppose \( p < 1 - F(\omega_v) \), then from Lemma 1 part (b.i) it follows that the ex-ante welfare of the DM is given by

\[
U(\emptyset, v) = F(\omega_v)\zeta + (1 - F(\omega_v))\tau
\]  

(10)

In the presence of a media it follows from Lemma 3 part (b.i) that if \( p < \frac{1}{\sqrt{2}} \), then in the most influential informative equilibrium, the ex-ante welfare of the DM is given by equation (8). From equations (8) and (10) we have \( U(m_k, v) > U(\emptyset, v) \) iff \( F(\omega_v)(\tau - \zeta)(2p - 1) > 0 \), which always holds since \( F(\omega_v) > 0, \tau > \zeta \) and \( p > 1/2 \). This proves the first case of part (b.ii) the proposition.

Suppose \( p < 1 - F(\omega_v) \), and \( p > \frac{1}{\sqrt{2}} \), then from Lemma 3 part (b.ii) it follows that under media the welfare of the DM is

\[
U(m_k, v) = \left( \frac{F(\omega_v)}{1 - p} \right) (p\tau + (1 - p)\zeta) + \left( 1 - \frac{F(\omega_v)}{1 - p} \right) \tau = F(\omega_v)\zeta + (1 - F(\omega_v))\tau
\]

Since comparing above equation with equation (10) we find that \( U(\emptyset, v) = U(m_k, v) \), this proves the second case of part (b.ii) of the proposition and completes the proof.

**Proof of Lemma 4 :**

Consider the case where a separating voting strategy is followed by all \( j \in I, j \neq i \).

Let \( E(n - 1, k) \) be the event that out of \( n - 1 \) private signals, exactly \( k \) equal \( X, k = 0, \cdots, n - 1 \). Voter \( i \) is pivotal if and only if \( k = \frac{n-1}{2} \). We shall use the shorthand \( Piv_i = E(n - 1, \frac{n-1}{2}) \). Let

\[
\tilde{\gamma}_{s_i} = P[\omega \leq \omega_v | Piv_i, s_i] = \frac{A}{B}
\]
where

\[ A = \Pr[\text{Piv}_i, s_i | \omega \leq \omega_v] \Pr[\omega \leq \omega_v] \]

\[ = \Pr[\text{Piv}_i | \omega \leq \omega_v] \Pr[s_i | \omega \leq \omega_v] \Pr[\omega \leq \omega_v] \]

and

\[ B = \Pr[\text{Piv}_i, s_i | \omega \leq \omega_v] \Pr[\omega \leq \omega_v] + \Pr[\text{Piv}_i, s_i | \omega_v < \omega] \Pr[\omega_v < \omega] \]

\[ = \Pr[\text{Piv}_i | \omega \leq \omega_v] \Pr[s_i | \omega \leq \omega_v] \Pr[\omega \leq \omega_v] + \Pr[\text{Piv}_i | \omega_v < \omega] \Pr[s_i | \omega_v < \omega] \Pr[\omega_v < \omega] \]

Note that

\[ \Pr[\text{Piv}_i | \omega \leq \omega_v] = \Pr[\text{Piv}_i | \omega > \omega_v] = \left( \frac{n-1}{n-1} \right) p^{n-1} (1-p)^{n-1} \]

Given the prior density \( f(\omega) \) with the associated distribution \( F(\omega) \),

\[ \tilde{\gamma}_X = \frac{pF(\omega_v)}{pF(\omega_v) + (1-p)(1-F(\omega_v))} \]

and

\[ \tilde{\gamma}_Y = \frac{(1-p)F(\omega_v)}{(1-p)F(\omega_v) + p(1-F(\omega_v))} . \]

Note that under the simple majoritarian aggregation rule, \( \tilde{\gamma}_{s_i} = \gamma_{s_i} \), where \( \gamma_{s_i} = \Pr[\omega \leq \omega_v | s_i] \) is defined in the proof of Lemma 1. Fix any symmetric voting strategy profile \( v_{-i} \). Define

\[ U_X(\emptyset, v_{-i}, s_i) = \sum_{k=0}^{n-1} \left[ \Pr[E(n-1, k) | s_i] \left( \int_{\omega \in \Omega} u(\delta(v_{-i}, v_i = X), w) f(\omega | s_i) \left. d\omega \right) \right] \],

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and
\[
U_Y(\emptyset, v_{-i}, s_i) = \sum_{k=0}^{n-1} \left[ P[E(n-1, k)|s_i] \left( \int_{\omega \in \Omega} u(v_{-i}, v_i = Y, w)f(\omega|s) \right) d\omega \right].
\]

At this voting strategy profile \(v_{-i}\), voter \(i\) votes for \(X\) if and only if \(U_X(\emptyset, v_{-i}, s_i) > U_Y(\emptyset, v_{-i}, s_i)\). This reduces to
\[
(2\tilde{\gamma}_{s_i} - 1)\tau > (2\tilde{\gamma}_{s_i} - 1)\zeta.
\]

Since \(\tau > \zeta\), the above inequality holds if and only if \((2\tilde{\gamma}_{s_i} - 1) > 0\), which implies \(\tilde{\gamma}_{s_i} > 1/2\).

Hence for a separating strategy profile \(v\) where \(v_i(\emptyset, X) = X\) and \(v_i(\emptyset, Y) = Y\) for any \(i \in I\) to hold in equilibrium, both the conditions \(\tilde{\gamma}_X > 1/2\) and \(\tilde{\gamma}_Y < 1/2\) need to be satisfied.

Now consider the scenario where either of the condition \(\tilde{\gamma}_X > 1/2\) and \(\tilde{\gamma}_Y < 1/2\) is violated, in which case it follows that a separating voting strategy cannot be sustained in a symmetric equilibrium. In this case we consider other possible symmetric equilibria which are (i) \(v_i = X\) for each \(s_i \in S\) for all \(i \in I\) or (ii) \(v_i = Y\) for each \(s_i \in S\) for all \(i \in I\). Note that if for all \(j \in I, j \neq i\), the voting strategy \(v_j = X\) for each \(s_j \in S\) is followed, then voter \(i\) is never pivotal. Furthermore, for this case \(\Pr(w < w_i|v_{-i}, s_i) = \Pr(w < w_i|s_i)\). In an informative equilibrium, since the preference of the voter is \(X > Y\) if \(\omega \leq \omega_i\) and \(Y > X\) if \(\omega > \omega_i\), upon receiving a private signal \(s_i = X\) the voter \(i\) votes \(v_i = X\) if \(\gamma_X > 1/2\) and \(v_i = Y\) if \(\gamma_X < 1/2\). Similarly, upon receiving a private signal \(s_i = Y\) the voter \(i\) votes \(v_i = X\) if \(\gamma_Y > 1/2\) and \(v_i = Y\) if \(\gamma_Y < 1/2\). These two observations along with the fact that \(\tilde{\gamma}_{s_i} = \gamma_{s_i}\) completes the proof.

We now state and prove the following Claim:

**Claim 1.** Let \(J(n,p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (p)^j (1-p)^{n-j}\). Then (i) \(J(n,p)\) is increasing.
in $n$.

**Proof of Claim 1:**
Following Proposition 1 in Karotkin and Paroush (2003), we may express

$$J(n + 2, p) - J(n, p) = \left( \frac{n}{n-1} \right) p^{\frac{n+1}{2}} (1-p)^{n-\frac{n+1}{2}} p^2 \left( \frac{1-p}{p} \right)^2 \left( \frac{1}{n} - (\frac{1-p}{p})^2 \right)$$

$$= \left( \frac{n}{n-1} \right) p^{\frac{n+5}{2}} (1-p)^{\frac{n-1}{2}} \left( \frac{1-p}{p} \right)^2 \left( \frac{1}{n} - (\frac{1-p}{p})^2 \right)$$

Hence the sufficient condition for $J(n + 2, p) - J(n, p) > 0$ is $\frac{1-p}{p} - (\frac{1-p}{p})^2 > 0$, which holds in our model since $p \in (1/2, 1)$. This proves the claim.

**Proof of Lemma 5:**
Consider $1/2 < p < F(\omega_v)$. The result follows directly from Lemma 1 case (a.ii) and Lemma 4.

Let $1/2 < F(\omega_v) < p$. Note that $\mathbb{P}[\omega < \omega_v|s_i, m_k, Piv_i] = \mathbb{P}[\omega < \omega_v|s_i, m_k]$ for all $k$. Consider the Class 1 and Class 2 message strategies defined in the proof of Lemma 2. It follows from the proof of Lemma 2 that the optimum value of ex-ante payoff of the media under Class 1 message strategies is given by equation (1). The ex-ante payoff of the media under equilibria in Class 2 is given by

$$\mathbb{E}[u_m]_2 = \int_0^\alpha \tau_m f(\omega) d\omega + \int_\alpha^{\omega_v} (J(n, p)\tau_m + (1 - J(n, p))\zeta_m) f(\omega) d\omega + \int_{\omega_v}^\beta ((1 - J(n, p))\tau_m + J(n, p)\zeta_m) f(\omega) d\omega + \int_\beta^1 \zeta_m f(\omega) d\omega$$

where $J(n, p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (p)^j (1-p)^{n-j}$, which is simplified as

$$\mathbb{E}[u_m]_2 = (F(\alpha) + F(\beta))(1 - J(n, p))(\tau_m - \zeta_m) + F(\omega_v)(1 - 2J(n, p))(\zeta_m - \tau_m) + \zeta_m$$

(11)
Since $\tau_m > \zeta_m$ and $0 < J(n,p) < 1$, this implies that in order to maximize the ex-ante payoff of the media ($F(\alpha) + F(\beta)$) needs to be maximized subject to: (i) $\frac{p}{1-p}(F(\omega_v) - F(\alpha)) + F(\omega_v) > F(\beta)$, (ii) $\frac{1-p}{p}(F(\omega_v) - F(\alpha)) + F(\omega_v) < F(\beta)$, (iii) $0 \geq F(\alpha) < F(\omega_v)$, and (iv) $F(\omega_v) < F(\beta) \leq 1$. We denote this optimization problem as (**). Note that the optimization problems (** and (*)), the latter defined in the proof of Lemma 2 are identical. Hence analogous to the proof of Lemma 2 it follows that the optimum value of $\beta$ is given by the relation $F(\beta^{**}) = 1$, which implies $\beta^{**} = 1$, and the optimal $\alpha^{**}$ must satisfy the condition $F(\alpha^{**}) = \frac{F(\omega_v)}{p} - \frac{1-p}{p}$. Replacing the optimum values of $\alpha^{**}$ and $\beta^{**}$ in (11), we have the maximum ex-ante payoff of the media under equilibria belonging to Class 2 to be

$$E[u_m]^{**} = \left(\frac{F(\omega_v)}{p} - \frac{1-p}{p} + 1\right)(1-J(n,p))(\tau_m-\zeta_m) + F(\omega_v)(1-2J(n,p))(\zeta_m-\tau_m) + \zeta_m$$

(12)

After having described the equilibria which corresponds to the highest ex-ante payoff of the media in Class 1 and Class 2, we compare the ex-ante payoff of the media between these two equilibria. From (1) and (12) it follows that

$$E[u_m]^{**}_2 - E[u_m]^{**}_1 = \frac{V}{p}$$

(13)

where $V = (\tau_m - \zeta_m)(J(n,p)(F(\omega_v) - 1)(2p - 1) + p(2 - F(\omega_v)) - 1)$.

We now show that $E[u_m]^{**}_2 - E[u_m]^{**}_1 < 0$ holds for all $n \geq 3$, which is proved if the following (14) can be shown to hold for all $n \geq 3$

$$J(n,p)(F(\omega_v) - 1)(2p - 1) + p(2 - F(\omega_v)) - 1 < 0$$

(14)

Let $K(n,p,F(\omega_v)) = J(n,p)(F(\omega_v) - 1)(2p - 1) + p(2 - F(\omega_v)) - 1$. Note that since $F(\omega_v) < 1$ and $p > 1/2$, we have $\frac{\partial K(n,p,F(\omega_v))}{\partial J(n,p)} < 0$. From Claim 1 it follows that if (14) can be shown to hold for $n = 3$, it will hold for $n > 3$. Note that when
$n = 3$, we have $J(3, p) = 3p^2(1 - p) + p^3$. Hence we have

$$K(3, p, F(\omega_v)) = 4p^4(1 - F(w_p)) + 8p^3(F(\omega_v) - 1) + 3p^2(1 - F(w_p)) + p(2 - F(\omega_v)) - 1$$

Note that $\frac{\partial K(3, p, F(\omega_v))}{\partial F(\omega_v)} = -p(4p^3 - 8p^2 + 3p + 1)$. Also note that $\frac{\partial K(3, p, F(\omega_v))}{\partial F(\omega_v)}|_{p=1/2} = -1/2$, $\frac{\partial K(3, p, F(\omega_v))}{\partial F(\omega_v)}|_{p=1} = 0$, and $\frac{\partial^2 K(3, p, F(\omega_v))}{\partial p^2} = 0$ has no solution in $p \in (1/2, 1)$. Hence $\frac{\partial K(3, p, F(\omega_v))}{\partial F(\omega_v)} < 0$ for all $p \in (1/2, 1)$. Therefore the maximum value of $K(3, p, F(\omega_v))$ is attained at $K(3, p, 1/2) = 2p^4 - 4p^3 + \frac{3p^2}{2} + \frac{3p}{2} - 1$. Now $K(3, p, 1/2)|_{p=1/2} = -\frac{1}{4}$, $K(3, p, 1/2)|_{p=1} = 0$, and $\frac{\partial K(3, p, 1/2)}{\partial p} = 0$ has no solution in $p \in (1/2, 1)$. Hence $K(3, p, F(\omega_v)) < 0$ for all $p \in (1/2, 1)$, $F(\omega_v) \in (1/2, 1)$.

We now argue that the optimum equilibrium obtained under Class 1 corresponds to the maximum ex-ante payoff of the media among all possible equilibria. Note that under Class 1 equilibria, $\mathbb{E}[u_m]$ is increasing in $\hat{w}$, and the maximum value of $\hat{w}$ under Class 1 is when the condition $F(\hat{w}) = \frac{F(\omega_v)}{p}$ is satisfied. When $F(\hat{w}) > \frac{F(\omega_v)}{p}$ then $\gamma < 1/2$ and we revert to equilibria under Class 2 with $\alpha = 0$, $\beta = \hat{w}$ which has been proved to have a lower expected payoff for the media than the optimum Class 1 equilibrium. This concludes the proof.

Proof of Lemma 6 : Since $\mathbb{P}[\omega < \omega_v|s_i, m_k, Pivot] = \mathbb{P}[\omega < \omega_v|s_i, m_k]$ for all $k$, it follows from the proof of Lemma 2 that the optimum value of ex-ante payoff of the media, denoted by $\mathbb{E}[u_m]|^*$ under Class 1 message strategies (as defined in the proof of Lemma 2) is given by (1). Consider Class 2 message strategies as defined in the proof of Lemma 2. From the proof of Lemma 5 it follows that the message strategy which maximizes the ex-ante payoff of the speaker is obtained by solving the optimization problem (**).

Now consider $F(\omega_v) < 1/2$, and $p > 1 - F(\omega_v)$, which implies $1 < \frac{F(\omega_v)}{1 - p}$. In this case the solution of (**) is given by $(\alpha^*, \beta^*)$ such that $F(\beta^*) = 1$ and $F(\alpha^*) = \frac{F(\omega_v)}{p} - \frac{1 - p}{p}$. Replacing the optimum values of $\alpha^*$ and $\beta^*$ in (11), we have the maximum ex-ante payoff of the speaker under Class 2 message strategies
to be given by (12). Parts (a.i) and (a.ii) of this lemma therefore follows from (14).

Now consider $F(\omega) < 1/2$, and $p < 1 - F(\omega)$, which implies $\frac{F(\omega)}{1-p} < 1$. In this case the solution of \((**\)\) is given by $(\alpha^{**}, \beta^{**})$ such that $F(\alpha^{**}) = 0$, and $F(\beta^{**}) = \frac{F(\omega)}{1-p}$. Replacing the optimum values of $\alpha^{**}$ and $\beta^{**}$ in (11), we have the maximum ex-ante payoff of the media under equilibria belonging to Class 2 to be

$$E[u_{m}]^{**}_{2} = (\frac{F(\omega)}{1-p})(1-J(n,p))(\tau_{m} - \zeta_{m}) + F(\omega)(1-2J(n,p))(\zeta_{m} - \tau_{m}) + \zeta_{m} \quad (15)$$

From (1) and (15) it follows that

$$E[u_{m}]^{**}_{1} - E[u_{m}]^{**}_{2} = \frac{D}{p(1-p)}$$

where $D(\omega, n, p) = F(\omega)(\tau_{m} - \zeta_{m})(J(n,p)p(2p-1) - p^{2} - p + 1)$.

Let $\eta(n, p) = J(n,p)p(2p-1) - p^{2} - p + 1$. Since $p > 1/2$, therefore $\eta(n, p)$ is increasing in $J(n, p)$, which by Claim 1 is increasing in $n$. Note that $\eta(5, 1/2) = 1/4$, $\eta(5, 1) = 0$, and $\eta(5, p) = 0$ does not have a solution in $p \in (1/2, 1)$. Hence $\eta(5, p) > 0$ for all $p \in (1/2, 1)$, which implies $D(\omega, 5, p) > 0$ for all $p \in (1/2, 1)$.

Since $D(\omega, n, p)$ is increasing in $n$, this proves part (b.i) of the lemma.

To prove part (b.ii), consider $n = 3$. Note that $\eta(3, 1/2) = 1/4$, $\eta(3, 1) = 0$, and $\eta(3, p) = 0$ has a unique solution in $p \in (1/2, 1)$ given by $p' = \left(\frac{27 - 3\sqrt{78}}{6}\right)^{\frac{1}{2}} + \frac{13\sqrt{78} + 27}{6} \approx .76$. This shows that for all $p \in (1/2, p')$, we have $D(\omega, 3, p) > 0$ while for all $p \in (p', 1)$, we have $D(\omega, 3, p) < 0$. This proves part (b.ii) of the lemma and completes the proof.

**Proof of Proposition 2**:
Consider $1/2 < p < F(\omega)$. It follows from Lemma 1 part (a.ii), Lemma 4 and Lemma 5 that in equilibrium, for both the cases when the media is absent or
present, the voter welfare is given by

\[ U(\emptyset, v) = U(\Omega, v) = \int_{0}^{\omega_v} \tau f(\omega) d\omega + \int_{\omega_v}^{1} \zeta f(\omega) d\omega = F(\omega_v)\tau + (1 - F(\omega_v))\zeta \]

This proves part (a) of the proposition.

Consider \(1/2 < F(\omega_v) < p\). From Lemma 1 part (a.i) and Lemma 4 it follows that in the absence of a media the ex-ante voter welfare is given by

\[ U(\emptyset, v) = J(n, p)\tau + (1 - J(n, p))\zeta, \]

where

\[ J(n, p) = \sum_{j=n+1}^{n+1} \binom{n}{j} p^j(1-p)^{n-j}. \]

From Lemma 5 it follows that in the presence of a media, the most influential equilibrium \((m_k, v)\) results in the ex-ante voter welfare given by

\[ U(m_k, v) = \tau F(\omega_v) + \zeta \left( \frac{F(\omega_v)}{p} - F(\omega_v) \right) + \tau \left( 1 - \frac{F(\omega_v)}{p} \right). \]

Hence \(U(\emptyset, v) > U(m_k, v)\) if

\[ \frac{(\tau - \zeta)(J(n, p)p - p(F(\omega_v) + 1) + F(\omega_v))}{p} > 0, \]

which holds if the following condition (16) is satisfied.

\[ F(\omega_v) > \frac{p}{1-p} (1 - J(n, p)) = G(n, p). \quad (16) \]

We now state and prove two additional claims:

**Claim 2.** For all \(p\) below \((1/2)(1 + \frac{2}{n+1})\), any critical point of \(G(n, p)\) is a strict local maximum and any critical point above is a strict local minimum.
Proof of Claim 2: Let $L_{n,j}(p) \equiv \binom{n}{j}p^j(1-p)^{n-j}$. Also let

$$F_{n,r}(p) \equiv \sum_{j=0}^{r} \binom{n}{j}p^j(1-p)^{n-j} = \sum_{j=0}^{r} L_{n,j}(p).$$

Since $n$ is an odd integer, and let $m$ be an even integer given by $m = \frac{n+1}{2}$. We may express

$$G(n,p) \equiv \frac{p}{1-p}F_{n,m-1}(p)$$

At $p = 0$ we replace this definition by its limiting value $G(n,0) = 0$. We also have $G(n,1) = 0$. Note that

$$\frac{d}{dp} L_{n,j}(p) = n (L_{n-1,j-1} - L_{n-1,j}) \tag{17}$$

So

$$\frac{d}{dp} F_{n,m-1}(p) = \sum_{j=0}^{m-1} n (L_{n-1,j-1} - L_{n-1,j}) = -nL_{n-1,m-1}.$$ 

Hence we have

$$\frac{d}{dp} G(n,p) = \frac{1}{(1-p)^2} F_{n,m-1}(p) - \frac{np}{(1-p)} L_{n-1,m-1-1} \tag{18}$$

Differentiating both sides of (18) again with respect to $p$ we have

$$\frac{d^2}{dp^2} G(n,p) = \frac{2}{(1-p)^3} F_{n,m-1}(p) - \frac{2n}{(1-p)^2} L_{n-1,m-1} - n \left( \frac{1}{1-p} - 1 \right) L'_{n-1,m-1}.$$  

From (17) we have

$$L'_{n-1,m-1} = (n-1) (L_{n-2,m-2} - L_{n-2,m-1}).$$
and hence
\[
\frac{d^2}{dp^2} G(n,p) = \frac{2}{(1-p)^3} F_{n,m-1}(p) - \frac{2n}{(1-p)^2} L_{n-1,m-1} - n(n-1) \frac{p}{(1-p)} (L_{n-2,m-2} - L_{n-2,m-1})
\]  
\hspace{1cm} (19)

By the Weierstrass theorem, \( G(n,p) \) has at least one global maximum over \([0, 1]\).

We now require \( G'(n,p^*) = 0 \), so from (18) we have
\[
\frac{1}{(1-p^*)^2} F_{n,m-1}^* - \frac{np^*}{1-p^*} L_{n-1,m-1}^* = 0
\]
where \( F_{n,m-1}^* \equiv F_{n,m-1}(p^*) \) etc.

For maximization we require \( G''(n,p^*) \leq 0 \), so from (19) we have
\[
\frac{2}{(1-p^*)^3} F_{n,m-1}^* - \frac{2n}{(1-p^*)^2} L_{n-1,m-1}^* - n(n-1) \frac{p^*}{1-p^*} (L_{n-2,m-2}^* - L_{n-2,m-1}^*) \leq 0
\]

Using the first-order condition, the second order condition is equivalent to
\[-2n(1-p^*) L_{n-1,m-1}^* - n(n-1) p^* (1-p^*) (L_{n-2,m-2}^* - L_{n-2,m-1}^*) \leq 0,
\]
which is further simplified as, so the SOC is equivalent to
\[2 + (n-1)p^* \left( \frac{m-1}{n-1} \frac{1}{p^*} - \frac{n-m}{n-1} \frac{1}{1-p^*} \right) \geq 0
\]
\[\iff m + 1 \geq (n + 1)p^*.
\]

Using \( n = 2m - 1 \) we can write the above inequality as \( p^* \leq \frac{m+1}{2m} \), which implies that
\[p^* \leq (1/2) \left( 1 + \frac{1}{m} \right).
\]  
\hspace{1cm} (20)
Thus it follows from (20) that any critical point below \((1/2) \left(1 + \frac{1}{m}\right)\) is a strict local maximum and any critical point above is a strict local minimum. Putting \(m = \frac{n+1}{2}\) proves the claim.

Claim 3. \(\frac{dG(n,p)}{dp}|_{p=1/2} < 0\) for all \(n \geq 7\).

Proof of Claim 3: Note that

\[
\frac{dG(n,p)}{dp} = \frac{1}{(1-p)^2} - \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (p)^j (1-p)^{n-j} \left[ \frac{j+1}{1-p} - \left(\frac{p}{(1-p)^2}\right)(n-j-1) \right]
\]

Hence

\[
\frac{dG(n,p)}{dp}|_{p=1/2} = 4 \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j+2-n) \right]
\]

We want to establish that for \(n \geq 7\), \(\frac{dG(n,p)}{dp}|_{p=1/2} < 0\) which implies

\[
2^{n+1} < \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j+2-n) \quad (21)
\]

Note that \(\sum_{j=0}^{n} \binom{n}{j} = 2^n\), and since \(n\) is odd, we also have

\[
\sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} = (1/2) \sum_{j=0}^{n} \binom{n}{j}.
\]

Using these, we may express (21) as

\[
2^n < \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2j-n) \quad (22)
\]
Let $k = \frac{n-1}{2}$. We may express (22) as

$$2 \left[ \binom{2k+1}{k+1} + \binom{2k+1}{k+2} + \ldots \right] < \binom{2k+1}{k+1} + 3 \binom{2k+1}{k+2} + 5 \binom{2k+1}{k+3} + \ldots + (2k+1)$$

which holds if

$$2 \left[ \binom{2k+1}{k+1} + \binom{2k+1}{k+2} + \binom{2k+1}{k+3} \right] < \binom{2k+1}{k+1} + 3 \binom{2k+1}{k+2} + 5 \binom{2k+1}{k+3}$$

Using the relation

$$\binom{n}{j+1} = \frac{n-j}{j+1} \binom{n}{j},$$

the above inequality reduces to

$$\frac{k}{k+2} + \frac{3k(k-1)}{(k+2)(k+3)} > 1$$

Consider

$$g(k) = \frac{k}{k+2} + \frac{3k(k-1)}{(k+2)(k+3)} - 1.$$  

Note that for $k > 0$, the equation $g(k) = 0$ has a unique solution at $k = \sqrt{\frac{37}{6}} + \frac{5}{6} < 3$, and $\frac{dg(k)}{dk}_{k=\sqrt{\frac{37}{6}} + \frac{5}{6}} > 0$. Hence $g(k) > 0$ for all $k \geq 3$, which implies (22) is satisfied for all $n \geq 7$. This proves Claim 3.

Now, note that when $p = 1/2$, then $J(n,1/2) = 1/2$ for all $n$ and hence $G(n,1/2) = 1/2$. From Claim 2 we know that for the function $G(n,p)$ any critical point when $p$ is below $(1/2)(1 + \frac{2}{n+1})$ must correspond to a *strict* local maximum and therefore cannot be a local minimum. But from Claim 3 we know that when $n \geq 7$, then $\frac{dG(n,p)}{dp}_{p=1/2} < 0$, and hence from Claim 2 it follows that there does not exist any $p \in (1/2, (1/2)(1 + \frac{2}{n+1}))$ such that $G(n,p) \geq 1/2$. From Claim 2 it follows that when $n \geq 7$, any critical value of $G(n,p)$ for $p$ above $(1/2)(1 + \frac{2}{n+1})$
must correspond to a strict local minimum. Since \( G(n, 1) = 0 \), it must therefore be that \( G(n, p) < 1/2 \) when \( p \in [(1/2)(1 + \frac{2}{n+1}), 1) \). Therefore we have shown that when \( n \geq 7 \), \( G(n, p) < 1/2 \) for all \( p \in (1/2, 1) \).

Since \( F(\omega_v) \in (1/2, 1) \), it follows that when \( n \geq 7 \), the condition \( F(\omega_v) > G(n, p) \) holds for all \( p \in (1/2, 1) \) and therefore from inequality (16) we have \( U(\emptyset, v) > U(m_k, v) \). This proves part (b.i) of the proposition.

To prove part (b.ii), consider the case when \( n \in \{3, 5\} \).

Let \( n = 3 \). Note that \( G(3, 1/2) = 1/2 \), \( \frac{dG(3, p)}{dp}\big|_{p=1/2} = 1/2 > 0 \), and that in the range \( p \in (1/2, 1) \), the equation \( \frac{dG(3, p)}{dp} = -6p^2 + 2p + 1 = 0 \) yields a unique solution given by \( p_3^* = \frac{\sqrt{7}}{6} + \frac{1}{6} > 1/2 \). Also note that \( \frac{d^2G(3, p)}{dp^2} = -12p + 2 < 0 \) for all \( p \in (1/2, 1) \). Hence the maximum value of \( G(3, p) \) is \( G(3, p_3^*) = \frac{2\sqrt{7}}{34} + \frac{5}{27} = k^*(3) \) which is greater than 1/2 and less than 1. Therefore it follows from (16) that when \( F(\omega_v) > k^*(3) \), then ex-ante voter welfare is always higher in the absence of a media. This proves part (b.ii.1) of the proposition for \( n = 3 \).

Suppose \( 1/2 < F(\omega_v) < k^*(3) \). Since \( \frac{d^2G(3, p)}{dp^2} < 0 \) for all \( p \in [1/2, 1] \), it follows that there exists \( F(\omega_v) < \hat{p}(3) < p_3^* \) such that for all \( p \in (F(\omega_v), \hat{p}(3)) \), the inequality \( G(3, p) < F(\omega_v) \) holds while for all \( p \in (\hat{p}(3), p_3^*) \), the inequality \( G(3, p) > F(\omega_v) \) holds. Since \( G(3, p_3^*) > 1/2 \) and \( G(3, 1) = 0 \) it follows that there exists \( p_3^* < \hat{p}(3) \) such that for all \( p \in [p_3^*, \hat{p}(3)] \), the inequality \( G(3, p) > F(\omega_v) \) holds while for all \( p \in (\hat{p}(3), 1) \), the inequality \( G(3, p) < F(\omega_v) \) holds. This proves part (b.ii.2) of the proposition for \( n = 3 \).

Now consider \( n = 5 \). Note that \( G(5, 1/2) = 1/2 \), \( \frac{dG(5, p)}{dp}\big|_{p=1/2} = \frac{1}{8} > 0 \), and that in the range \( p \in (1/2, 1) \), the equation \( \frac{dG(5, p)}{dp} = 30p^4 - 36p^2 + 3p^2 + 2p + 1 = 0 \) yields a unique solution given by \( p_5^* = \frac{(548 - 30\sqrt{290})^{\frac{1}{4}}}{30} + \frac{(30\sqrt{290} + 548)^{\frac{1}{4}}}{30} + \frac{1}{15} \). Note that \( 1/2 < p_5^* < p_3^* \) and \( 1/2 < G(5, p_5^*) < G(3, p_3^*) < 1 \). Let \( G(5, p_5^*) = k^*(5) \). When \( F(\omega_v) > k^*(5) \), it follows from inequality (16) that ex-ante voter welfare is higher in the absence of a media. This proves part (b.ii.1) of the proposition for \( n = 5 \).

Suppose \( 1/2 < F(\omega_v) < k^*(5) \). Since \( \frac{d^2G(5, p)}{dp^2}\big|_{p=1/2} < 0 \), \( \frac{d^2G(5, p)}{dp^2}\big|_{p=p_5^*} < 0 \), and the equation \( \frac{d^2G(5, p)}{dp^2} = 0 \) is not solved for \( p \in [1/2, p_5^*] \), it follows that \( \frac{d^2G(5, p)}{dp^2} < 0 \).
for all $p \in [1/2, p_5^*]$. This proves the existence of $\hat{p}(5)$. Since $G(5, p_5^*) > 1/2$ and $G(5, 1) = 0$, the existence of $\hat{p}(5)$ is proved. This proves part (b.ii.2) of the proposition for $n = 5$. This concludes the proof.

**Proof of Proposition 3:**
Let $1 - p < F(\omega_v) < \frac{1}{2}$. From Lemma 1 part (b.i) and Lemma 4 it follows that in this case the ex-ante welfare of the voter in the absence of the media is given by

$$U(\emptyset, v) = J(n, p) \tau + (1 - J(n, p)) \zeta \tag{23}$$

Now consider the case where the media is present. Suppose $(\ast)$ holds. From Lemma 6 part (a.i), the ex-ante welfare of the voter in this case is given by

$$U(m_k, v) = \tau F(\omega_v) + \zeta \left( \frac{F(\omega_v)}{p} - F(\omega_v) \right) + \tau \left( 1 - \frac{F(\omega_v)}{p} \right) \tag{24}$$

It follows that in this case $U(\emptyset, v) > U(m_k, v)$ if (16) holds.

Suppose $(\ast)$ is always violated, in which case by Lemma 6 part (a.ii) it follows that the ex-ante welfare of the voter is given by

$$U(m_k, v) = \tau \left( \frac{F(\omega_v)}{p} - \frac{1 - p}{p} \right) + (1 - \left( \frac{F(\omega_v)}{p} - \frac{1 - p}{p} \right))J(n, p) \tau + (1 - J(n, p)) \zeta .$$

Since $p > 1 - F(\omega_v)$, it follows that $0 < \frac{F(\omega_v)}{p} - \frac{1 - p}{p} < 1$ and since $\tau > \zeta$, $0 < J(n, p) < 1$ it follows that for this case $U(m_k, v) > U(\emptyset, v)$. This proves Part (a) of the proposition.

Now consider $F(\omega_v) < 1 - p$. It follows from Lemma 1 part (b.ii) and Lemma 4 that in this case the ex-ante welfare of the voter in the absence of a speaker is given by

$$U(\emptyset, v) = \int_0^{\omega_v} \zeta f(\omega)d\omega + \int_{\omega_v}^1 \tau f(\omega)d\omega = F(\omega_v) \zeta + (1 - F(\omega_v)) \tau$$

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From Lemma 6 part (b.i) it follows that in the presence of the media, when \( n \geq 5 \), the ex-ante welfare of the voter is given by equation (24). Hence it follows that \( U(m_k, v) > U(\emptyset, v) \) if

\[
(\tau - \zeta)(2F(\omega_v) - \frac{F(\omega_v)}{p}) > 0
\]

which always holds for all \( 1/2 < p < 1 \). Hence in this case the presence of the media leads to higher voter welfare. When \( n = 3 \), and \( p \in (1/2, p') \) where \( p' \) is defined in Lemma 6 part (b.ii), it is analogously shown that (25) holds and presence of the media leads to higher voter welfare. When \( n = 3 \), and \( p \in (p', 1) \), from Lemma 6 part (b.ii) it follows that under media presence, the ex-ante voter welfare is given by

\[
U(m_k, v) = \left( \frac{F(\omega_v)}{1-p} \right) (J(n, p)\tau + (1 - J(n, p))\zeta) + \left( 1 - \frac{F(\omega_v)}{1-p} \right) \tau.
\]

Hence it follows that \( U(m_k, v) > U(\emptyset, v) \) if

\[
\frac{F(\omega_v)(\tau - \zeta)(J(n, p) - p)}{1 - p} > 0
\]

which always holds since \( F(\omega_v) > 0, \tau > \zeta \) and \( 1/2 < p < J(n, p) \). Hence in this case the presence of media leads to higher ex-ante voter welfare. This proves part (b) of the proposition and concludes the proof.

7 References


Angeletos, G. M. and A Pavan (2004): “Transparency of Information and Co-


