How Do Mathematicians Make Sense of Definitions?

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With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon. (Ball, 1993)

The Plan

• Engagement
• Background of our work
• A surprise
• Emerging themes

Making Sense of Definitions

• What do you do to make sense of a new definition? How do you know when you understand a definition?
• What do you do to support students’ understanding of mathematical definitions? How does this compare to what you do for yourself?

Definitions Research

• Mathematical definitions are different (Edwards & Ward, 2004)
• Mathematicians’ views on key features of definitions (Harel, Selden, & Selden, 2006)
• Concept image versus concept definition (Tall & Vinner, 1981)
• Defining as a mathematical activity (Zandieh & Rasmussen, 2010)

Data Collection

• Methods:
  – Student surveys
  – Interviews of students (8) and mathematicians (8)
• Purpose:
  – Gain insight into the processes mathematicians use to make sense of new definitions
  – Compare mathematician responses to student responses
The Interview

- What helps you understand a new definition?
- How do you help students understand definitions?
- Example generation activity
- Definition task

Data Analysis

What I say I do

What I say I do for students

What I did

What I Say I Do

- Adam: “I immediately try to think of examples of what would satisfy the requirements of the definition and what not.”
- Sam: “The simplest thing to try first is just to look at the specific concrete examples.”
- Greg: “Examples.”

What I Say I Do For Students

- Adam: “I try to kind of simulate for them this process I usually go through when I try to understand something.”
- Sam: “Sometimes we just look at very concrete examples.”
- Greg: “Of course, plenty of examples and plenty of pictures.”

First Comparison

Definition Task

A formal language is the support of a formal power series over $X^*$ where $X$ is an alphabet.

(Salomaa & Soittola, 1978)
What I Did

- Decoding process
  - Specific words: asked for meanings, related to previous knowledge
  - Notation: questioned use of notation
  - Generality: questioned level of generality
- Unfinished process
  - Next steps

Second Comparison

Purpose of a Definition

Mathematicians’ processes for making sense of a definition necessarily involve considering the usefulness of the definition within a particular mathematical setting.

Purpose of a Definition

- Adam: “I’ve never thought about formal language in this way, this abstractly. It’s probably very useful; it’s probably very general.”
- Sam: “I’d like to see, when confronted with something like this, what kind of operations one is interested in doing with language.”
- Sadie: “I like to see the word in context, what it is being used for.”

Role of Examples

Mathematicians see examples as a multi-faceted tool for understanding definitions.

Facets?

- Building intuition
- Developing precision
- Using test cases
Building Intuition

- Adam: “So we started with these two groups and we first played a little bit with this one. [I] tried to lead them to some of the essential properties.”
- Sam: “Definitions come from isolating particular examples so that by the time you reach the definition, you have already acquired, hopefully, certain intuition about, about what this is… and then when you get to the definition, you see, okay, obviously these are the important features.”

Developing Precision

- Adam: “I don’t think a definition is something that one can claim to have really under control unless you can verify that certain mathematical objects satisfy or don’t satisfy the definition.”
- Greg: “Now I would have to, kind of like, … take an explicit alphabet and actually write down an example. I mean, because the importance, in particular, the importance of this, uh, semiring in relation with the whole construction, that’s something I would have to think about. What’s really the… or where do I need the ring structure, things like that.”

Using Test Cases

- Adam: “So then I found the definition and of course I immediately started checking if the real line has this property, if the set of integers with each addition operation has this property… And I started wondering what things don’t have these properties.”

Why do we offer students examples in class, and what are they supposed to make of them? If examples are always examples of something, how can students become aware of that which the examples are supposed to be exemplifying? (Mason & Pimm, 1984)

Next Steps

- Detailed analysis of example usage
- Further development of themes
- Analyze student responses, then compare with mathematician analysis
- Design follow-up study

Thank You!

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